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ABSTRACT

Thinking Approach to Problem Solving (TAPS) is a decision-making model that will assist students, kindergarten through grade eight, to decide on the choice of operation in routine mathematical word problems. The TAPS algorithm, or plan, suggests that students read a word problem, determine the question to be answered in the problem, use concrete manipulatives to model the groups, and analyze action(s) that must be taken with groups to answer the question(s) asked in the problem. After modeling the groups students can decide on the solution strategy. The thinking strategy begins with application of addition and subtraction operations. The multiplication and division operations apply to the middle grades. Included in the document are: chapter 1 (containing an introduction and sections on the thinking process, the communication process, and mathematical processes); chapter 2 (explaining the three basic actions and the TAPS Model); chapter 3 (providing a scope and sequence of TAPS applications and sample lessons); and a bibliography. (RH)

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TAPS - A THINKING APPROACH TO PROBLEM SOLVING

A Cognitive Schema for Choosing the Operation in Routine Mathematical Word Problems, Kindergarten through Grade Eight

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October, 1987

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Preface

TALTON'S TAPS (a Thinking Approach to Problem Solving) is a decision making model that will assist students, kindergarten to grade eight, to decide on the choice of operation in routine mathematical word problems, frequently the most difficult step in the problem solving process. The TAPS algorithm, or plan, suggests that students read a word problem, determine the question to be answered in the problem, use concrete manipulatives to model the groups, and analyze the action(s) that must be taken with the groups to answer the question(s) asked in the problem. These actions include combine, compare, and separate. After modeling the groups, students can decide on the solution strategy by considering:

- (1) If there are 2 or more groups, or sets, in a word problem, the groups will be either *combined* or *compared*,
- (2) If there is just one group, or set, in the word problem, the group can only be *separated*.

This thinking strategy begins in the primary grades with application of only the addition and subtraction operations. Addition is used to combine 2 or more groups and subtraction is used to either compare 2 or more groups to find a difference, or to separate one group into parts. The multiplication and division operations apply in the middle grades as students analyze word problems in greater depth. Students learn that *equal* groups are combined using multiplication and that division is used to separate one group into *equal* parts. They further learn that division is used to compare groups to find a *ratio*.

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FOREWORD

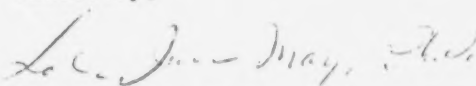
Is there a teacher alive that hasn't felt that teaching students to solve word problems is a hopeless task? Most teachers read many articles written about word problems. The end result has been that the majority of children dislike word problems and find them difficult and that teachers dislike teaching them. That is the bad news. Now, here is the good news! There is a method of teaching word problems that works for children of all ages. The methods developed by Dr. Carolyn Talton and Dr. Azalia Francis have a central theme that can be used in every grade level starting at kindergarten and going through high school. The theme is looking at the problem and thinking in terms of part, part, whole. From this one theme, children can be taught to tell which operation is needed to solve the word problem.

The method employs all the good aspects of teaching. The teacher is involved with good stories. Students are involved in acting out the story with objects. There is a sound progression from the concrete, to the semi-concrete, to the abstract symbols that record the problem. The teacher is helped at all stages with good materials and good activities to go along with the materials.

When I finished reading the material and then used it with some children in the Winnetka Public Schools, I felt like someone had lifted a weight off my back. One of the main trouble spots in teaching mathematics now had a solution.

Hopefully, you will find the method TAPS is your answer to teaching word problems regardless of your grade level.

Sincerely,



Lola J. May, Ph.D.

Mathematics Consultant

Winnetka Public Schools

TAPS - A THINKING APPROACH TO PROBLEM SOLVING

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CHAPTER ONE
RATIONALE AND INTRODUCTION TO TAPS

CHAPTER ONE

Introduction: Rationale

Solving word problems is one of the most frequently utilized mathematics skills in the life of a citizen in contemporary society, yet there is wide-spread agreement that students in the elementary and secondary schools in the United States have major dysfunction with this process. The 1979 National Assessment of Education Progress indicated that the primary source of difficulty for students was their inability to choose the appropriate operation when solving mathematics word problems. The researchers further stated that children learn too much of their arithmetic without understanding how to apply what they know in order to solve word problems. The population of the study on which these conclusions were based numbered 70,000 nine and thirteen year old children. The nine-year olds averaged sixty percent correct responses on multiplication examples, but only twenty-eight percent could correctly solve word problems requiring the product of a two-digit number to be multiplied by a one-digit number.

Thirteen-year olds in the study averaged seventy percent correct responses on division computation in isolation, but only forty percent when solving a word problem requiring the same skill. Findings of this study clearly indicate that children can perform computations, but that the processes do not make sense to them in ways which enable them to use their computation skills as problem solving tools. Herrington (1980) affirmed this finding and further concluded that the absence of logical thought processes was the source of student error in problem solving, rather than computation procedures.

Widespread research has been conducted relative to problem solving behavior. Polya (1973) developed a strategy for approaching a written word problem which had the students "Read, Plan, Solve, and Check." This basic plan has been modified slightly and used in textbooks, workbooks, and other instructional material. Other researchers continued to develop numerous problem solving strategies and structures with little replicable, permanent success (Talton, 1973; Knifong, 1977; Zweng, Turner, and Geraghty, 1979; Herrington, 1980; Carpenter, Hiebert, and Moser, 1981; Ballew and Cunningham, 1982; Suydam, 1982; and Talton and Francis, 1986). Studies such as these have failed to produce a logical reasoning approach for students to use when solving word problems.

The Second International Mathematics Study, 1981-1982, (1985) Investigated the mathematical understanding of approximately 7,000 eighth grade and 5,000 twelfth grade students in the countries of Belgium, Canada, England, Finland, France, Hong Kong, Hungary, Israel, Japan, Luxembourg, Netherlands, New Zealand, Nigeria, Scotland, Swaziland, Sweden, Thailand, and the United States. The study concluded that students in the United States lack a logical schema that would result in the selection of an appropriate solution strategy for use in the problem solving process. Students in these same grades in the other countries were far superior in problem solving and application ability (see Tables I and II).

Costa (1987) indicates that some people are unaware of their own thinking processes while they are problem solving. They, "...don't know, they are just doing it." Furthermore, he states that these individuals are likely to acknowledge that they have no processing plan for solving problems.

Crosswhite (1986) challenged arithmetic teachers to find a way to "...teach *more* mathematics and to teach *mathematics* more especially in the early grades and to teach it in a more supportive climate in the home, school, and society." Much of the current effective school research indicates that the initial, direct, teacher-led, introduction of a process must be highly motivational and

Table I

Score in Algebra and Calculus for Top Five Percent of
Twelfth Graders: 1981 - 82

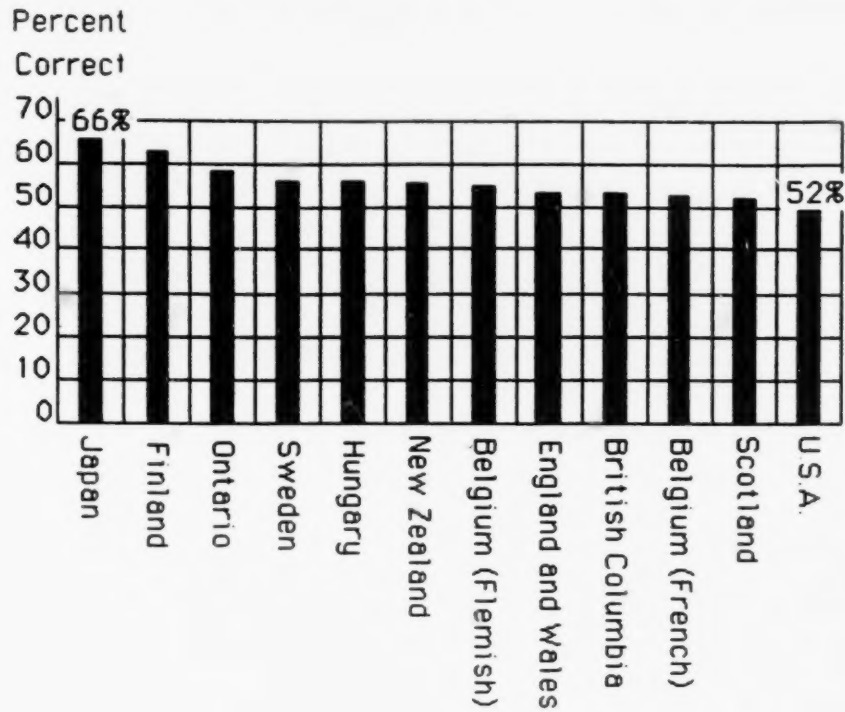
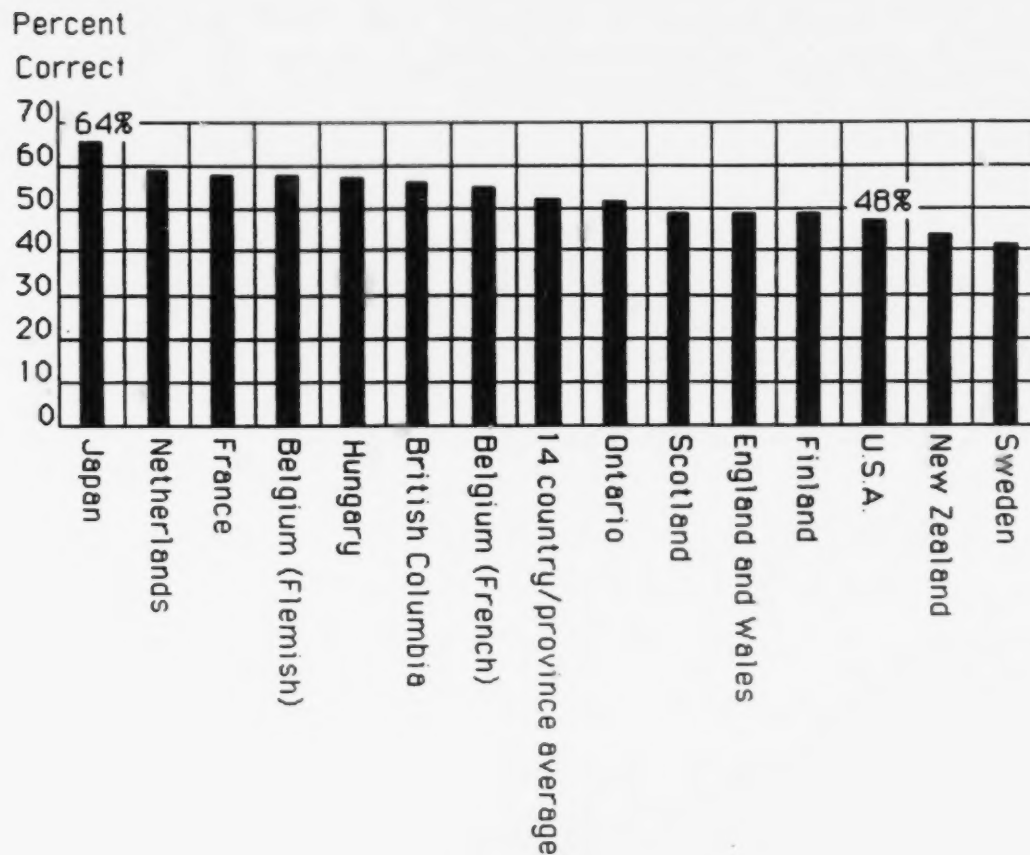


Table II
Average Mathematics Score for Students in
Eighth Grade: 1981 - 82



must engage the learners in verbal exchanges to assure clarity and precision of thought. Instruction should continue long enough and contain enough repetition to assure that the students acquire control of the knowledge which they then are to exhibit during the processing of the arithmetic information. "

Three basic, but crucial, areas of research must be examined. (1) The *nature of cognition*, particularly the levels of thought and the process of information reception and expression must be considered. (2) The *nature of communication* process must be examined. Although much research has been conducted about the impact of reading on a student's ability to solve problems, there is evidence that many students cannot solve problems even when the problem is read aloud to them. It would appear, then, that language barriers may hinder a student's ability to solve word problems. (3) Finally, the *nature of mathematics* as a discipline with algorithms, symbols, properties, operations, and relationships must be considered since problem solving without mathematical proficiency is of little value.

Briefly, each of these three concerns will be discussed in this chapter with reference to the following: thinking; listening and speaking; the mathematical processes and operations; the writing process; and reading. TAPS - A Thinking Approach to Problem Solving (Tolton and Francis, 1987), is detailed in Chapter Two. Sample model lessons for kindergarten through grade eight are found in Chapter Three. Appendix A contains the scope and sequence of the six important problem types included in the TAPS plan.

The Thinking Process

Much is being written today about the thinking process. The entire November, 1984, issue of Educational Leadership was devoted to teaching thinking skills. In that issue, Willoughby (1984, p. 90) states, "The ability to think mathematically is important to our collective and individual futures." He continues, "Leaders in mathematics education have been virtually unanimous in

calling for programs with a problem solving focus - programs that derive mathematics from practical experience and constantly relate it back to that experience. Such programs should include more and earlier work with data analysis, functions and graphs, estimation, geometry and measurement, probability, rational numbers - both common and decimal fractions, calculators and computers."

Although details of a teachable thinking process are not identified, the need was clearly stated by Costa (1987) in addressing the processes which lead to "Teaching for Intelligent Behavior." He indicates that there are three primary steps in logical thought: "input, processing, and output." In one of his presentations, Costa (1986) listed the behaviors associated with intelligent behavior: persistence, decreasing impulsivity, listening empathetically, flexibility, metacognition, checking for accuracy and precision, questioning and problem posing, drawing on past knowledge and applying it to new situations, precision of language and thought, multisensory awareness, creativity, and a sense of efficacy as a thinker." Instructional strategies and materials must include these processes if instruction in problem solving strategies is to be productive.

Benjamin Bloom (1956) identified levels of cognition in a hierarchy which moves from simple recognition and recall to the higher levels of application, analysis, synthesis, and evaluation. Arthur Costa (1986) has also suggested questions which teachers might pose to evaluate the degree to which their instruction promotes intelligent behavior. These questions include "Do teachers' responsive behaviors enhance and extend intelligent behaviors?" (p. 7) "Do students and teachers discuss their thinking?" (p. 9)

Much of the instructional material currently in use in the schools does little to teach a child beyond the application level of knowledge. Rarely is there a system deliberately designed to assure the development of patterns of

thought, that is , *schema* , which might be repeatedly used to solve problems.

Talton's Thinking Approach to Problem Solving (TAPS) contains a systematic process in which the student, under the direction of the teacher, moves from kindergarten through the eighth grade in a series of structured problem solving instructional activities which, summatively, assure the development of schema for a wide variety of problem situations using whole numbers, rational numbers, geometry, measurement, ratio and proportion, elementary statistics, and pre-algebra.

Deliberately, the student is led to identify three actions which are associated with one-step and multi-step word problems : *Combine* two or more groups, *Separate* one group into parts, and *Compare* two or more groups. These action terms are used with both whole and rational numbers in a variety of mathematical settings. Each step relates to the student manipulation of concrete objects under the direction of the teacher, following the demonstration of the skill process by the teacher. Movement from the concrete, to the pictorial, and finally to symbolic reasoning is equally well planned. Each child with each skill is deliberately taken up to the evaluative level of cognition, that of assessing the answer for accuracy.

The Communication Process: Listening and Speaking

Nesher and Katriel (1986) address the issue of learning numbers from a linguistic perspective. Although their research investigated primary students who did not comprehend the numeral "4" and the number word "four", confusion with mathematics in general was often found in problem solving instruction. They reported a knowledge that many teachers have observed when confronted with a written word problem: children simply lift the numbers from the written material and then appear to randomly use them. This behavior occurs because the *formal language* of mathematics is quite different from that of the child's home or

classroom. During the beginning stage of a mathematical process, the authors indicate that care must be taken to associate mathematics language with real objects to which numerical expressions apply.

Further, Nesher and Katriel suggest that at the end of the process, "after children have acquired the ability to operate with the formal language and have learned a number of operations and propositions in the language of mathematics,, we are interested in teaching them to use the mathematical models they have acquired in an unstructured, natural environment, an environment described in natural language terms." (p.100)

Nesher and Katriel further state that the two most common activities in the process of arithmetic instruction are basic operations with numbers in exercises and in word problems. In the conclusions for their study, the authors state that "Failing to understand the connection between the arithmetical operation to be performed and the text's story line" (p.110) can be a deterrent to solving word problems. "

Research on the school related language difficulties began approximately fifty years ago with particular attention being given to the relationship between spoken language and written language related to reading and writing activities. Little was done regarding the connection between language and mathematical word problem solving, yet it is obvious that there are many linguistic pitfalls for students. Several of these relate to regional, ethnic, or social dialects or mannerisms in the student's base language--the language of the child's thoughts.

While linguists agree that the base language of the child is valuable, there is also agreement concerning the difficulties encountered in the act of translation to school talk and to textbook written language. Selected features of language have been found by researchers to be particularly significant. Francis, (1970) identified selected features in the language commonly used in the southeastern

United States which place children at risk for reading comprehension, spelling, writing and mathematics instruction. These features include: (1) Improper enunciation of both regular and irregular plural words such as *boys* and *children*; (2) Omission of the final /s/ or /z/ on present tense verbs which hampers the student's awareness of the time of the action being considered; (3) Lack of differentiation between the forms of irregular verbs; and (4) Non-standard negation, such as "not none".

Care must be taken to assure that student language is appropriate for problem solving thought patterns, or schema. Through planned interaction - questioning and responding with corrective feedback as might be required - the student must be led to verbalize the process of problem solving, to identify possible language barriers, and to couple observation with conversation such that the words are generated which will ultimately allow the student to write problems for data recall or for clarification of complex problem situations and to solve problems which have been written by others.

Assumed language has been found by Francis (1986) to constitute another area of difficulty in student problem solving. Students have not traditionally been led to understand that in many problems there are unwritten, implied actions which must be comprehended if the student is to successfully solve the problem. Consider the following problem:

"Bill sells 49 small turtles and 39 large turtles. How many turtles did he sell in all?" (Houghton Mifflin Mathematics Program, Level 3, Teacher's Edition, Unit Test, p. 107).

The unstated, or assumed, action or situation in this problem could create a difficulty for the student. Suppose the student does not understand that the term *sell* is equal to "Bill has sold all of his turtles, both the big turtles and the little

turtles. This means that he does not have any turtles left." Only when the student receives this message will he be able to think about the operation of addition.

Communication: The Writing Process

Much is being said in the current literature about the value of having students record their thoughts and words in their own written expression. Rarely has the process been included as a portion of the instructional procedure to be utilized in teaching mathematical word problem solving. Simpson (1986) described a system for studying in the content areas. Although his immediate audience and instructional purpose were different from the mathematical problem solving outcome associated with this paper, the impact on students is thought to be the same. After four years Simpson found that utilizing a learning strategy identified as PORPE (Predict, Organize, Rehearse, Practice, and Evaluate), resulted in students who were, "actively planning, monitoring, and evaluating their learning of content, reorganizing information into their own words and schema, self questioning, and self testing. This procedure moves from total teacher guidance and instruction to total student independence and control.

Many of the descriptors given by Simpson were also used by Costa (1986) to describe a human being who was behaving intellectually. During the writing process each student must be led to understand the essential steps in the problem solving process, the mathematical language within the stated relationships and the implied language. Thus, when they use the writing process to create their own mathematics word problems, they have the same obligation to their reader.

The writing process recommended by the authors includes the following sequence of instruction: *Modeling, Discussing, Analyzing, Practicing, Draft Problem Writing, Self and Peer Evaluation, Revision, and Publication. Modeling* by the teacher with discussion between the teacher and the students about the

concrete representation of the problem, the pictorial representation, the symbolic representation, and then the mathematics language representation is essential in the problem solving sequence. This process is followed by student creation of oral examples of word problems which relate to the problem that was used by the teacher in the initial instruction and to the manipulatives being used by the students. For example, if the problem concerns ants, then plastic ants are used on the plate of the overhead projector to demonstrate the sample problems. This procedure relates to the concerns expressed by Nesher and Katriel (1986) and appears to the authors to be absolutely essential in the student attainment of successful problem solving behaviors including both the use of natural language and the use of mathematics language.

Discussion between the teacher and the student about the problem under consideration must be comprehensive and should include some participation by each child in the classroom. Crosswhite (1986) in contrasting time-use by the teachers in the United States with their counterparts in countries like Japan and France reported that a large percentage of teachers in the United States (U.S.) spend one class period talking about a range of topics, while the teachers in the other countries spend up to one hour, an entire class period, teaching a single problem with great thoroughness and intensity. He states that U.S. teachers spend time giving directions, that students "paid attention in class on average of less than half the time and that only one hour per week on the average was spent by U.S. teachers giving information *about* mathematics." The authors believe that this time must be expanded to the extent that each group of students each day have an opportunity to *analyze* mathematical problems.

Analysis includes the process of determining the number of groups, understanding the nature of the question of the problem, extracting the implied action in the problem, and selecting the mathematical operation necessary to address the question. This should be deliberate and thorough.

Following extensive modeling and process analysis, the students should *Practice* creating problems of their own which are read only by the teacher before they are given practice opportunities in which they construct draft written problems. *Drafting* is a process in which each student should be assured that this is part of the *learning process*, not *evaluation*. Using index cards, which denote "throw away after use", aides the students to comprehend and accept the drafting process as a vital step in the acquisition of problem solving skills. The draft problems should then be exchanged and read by peers in the classroom. If there is a communication deficit, it should be noted. If the message is unclear or if information is missing, the author should be given assistance in generating the final copy. This is the *Revising* stage of the writing process for problem solving skill development. Care should be taken that each problem is clearly stated, punctuated correctly, written legibly, and that the words are spelled correctly.

In the *Publishing* stage which follows, the students are to use their written problems in a public place. Often two teachers of the same grade level might choose to exchange the problems so developed. Some teachers plan to save the problems written by the students one year for use in the sequential years. Students seem to value becoming a part of a continuing process. Creation of a problem solving notebook for class use during spare time is a good idea. In many classrooms, the teacher will choose to create learning centers and interactive bulletin boards which display written mathematical word problems. This is an excellent source for publication of student problems.

Finally, the students should select some of their best examples to take to their homes for sharing with their parents and other family members. One excellent, but rare, use is a problem page in a small town, local newspaper. A teacher found the editor to favor the process of educational sharing and the readers indicated that they looked forward to seeing the multistage problems that the sixth grade students created.

Communication: The Reading Process

Reading comprehension is absolutely essential if students are to be able to solve textbook word problems in arithmetic. But more importantly, students must be able to read in life to solve the problems wherever and whenever they are presented. Often the written presentation is far more obtuse in real life than in school textbooks. For example, the person purchasing a bag of flour at the grocery might observe one unit price sign which stated, "16 ounces of XXX Flour, \$.07 per ounce" and another sign, "22 ounces of YY Flour, \$.95 per 1/2 pound". The consumer must select the flour to be purchased based on cost per ounce or cost per pound, if the quality of the goods is thought to be equal. To comprehend the problem requires considerable vocabulary skill, definitions of ounces and pound, as well as the mathematics vocabulary of "22" and "1/2". Additionally, the student must formulate an implied question. "Which of the two bags of flour is the better purchase?" The event must occur rapidly in time and result in an accurate number sentence.

Selected reading skills enter into the accomplishment of this skill and must be consciously developed during instructional procedures. *Drawing inferences* is essential if students are to comprehend the question of the problem. The skill of extrapolating the *main idea* from a written message must be mastered. Understanding the strategy of identifying *related details* is necessary for problem solving to be successful. *Sequential* thinking is mandated both in the problem solving process and in the multiple-step problem solution. *Summarizing* is required as the student translates a written message into a simple number sentence. *Comparison* is needed if the student is to assess his answer through the check method.

Sanacore (1985) has suggested instructional procedures which should occur if students are to acquire these reading behaviors. He groups these behaviors into

three categories: metacognitive discussions, strategy discussions, and post reading discussions. Several instructional activities which Sanacore presents enable the teacher to help the students attain these skills including: drawing or finding pictures that enrich the meaning of the text; engaging in prereading discussions that help children weave new information into old information; teaching students to self question; and linking reading and writing.

All these communication skills must be imbedded into the problem solving process such that a thinking, language-based approach to the instruction is consciously utilized by the teacher involved.

Mathematical Processes

In addition to language, mathematical principles must be carefully considered so that students do not develop the "guess and test" syndrome. In interviewing a group of graduate students, Talton and Francis (1986) found that, not only were these students afraid of written mathematics problems, they verbalized their tendency to just "Guess and get it done" as their mode of operation. There was no evidence among the graduate students, all of whom were elementary school teachers, that they knew of the possibility of a problem solving strategy which might be used repeatedly in different situations. Costa (1986) also identified this reactionary behavior with reference to problem solving assignments.

Talton (1984) identified and Bourgeois affirmed (1987) three actions which are associated with up to 96 percent of the word problems found in the state adopted mathematics textbooks in the state of Louisiana. *Combining* was found to be the action related to adding or multiplying two or more sets which were unequal or equal in number. *Separating* was the action which occurred when in the original situation there was one group which was to be separated using the

operation of subtraction or division. *Comparing* was the action that was identified with two or more groups which required subtraction or division to find differences or ratios between groups. Problem solving approaches which fail to consider the nature of mathematics cannot assist the students in their development of this essential behavior.

The research described in the preceding pages forms the rational base for the Thinking Approach to Problem Solving (TAPS). The TAPS plan is simple yet comprehensive and fits into the philosophy of the effective school research. The procedures are teacher-directed and involve whole-class groups during the initial instructional process. Guided practice is conducted with corrective feedback provided when the monitoring of student behavior indicates a need for alteration and/or clarification. Independent practice is also provided such that students are given opportunities to extend their skills and to maintain them as well.

The following example lesson plan will enable the reader to understand the TAPS process:

Teacher Objective:

Given the addends 5 plus 3 in a problem statement, each student will be able to correctly add these to obtain a sum of 8.

Teacher Guidelines:

The teacher places a transparency on the plate of the overhead projector and turns the equipment on. On the transparency is a simple line drawing of a yard in which a fence, a doorstep, a tree, and a garage door can be seen.

Reaching into a container, the teacher withdraws eight small, inexpensive plastic cat models. He/she invites the students to do the same with their plastic cats and their Situation Pages. For a few minutes, the teacher places the cats randomly on the transparency saying something like, "I would like for five of my cats to be on the fence and three on the doorstep. How would you like to place your cats?"

Various children might be given opportunities to display the nine different two-addend combinations for the value of eight. One or two of them might identify the multiple addend idea and would say, "I want to group my cats in three sets. One group is in the tree, another on the fence, and another in the garage. Much flexibility should be allowed.

The teacher should then read the problem:

There were five cats under a tree. Three cats were on the doorstep. The dog next door barked scaring the cats and they all jumped up on the fence. How many cats are on the fence?

The teacher should carefully display the model cats using the transparency as he/she tells the story problem.

The teacher should repeat the story asking the students to make their Situation Pages look like the transparency. This modeling process tends to assist students to avoid haphazard guessing. When the situation is in place and the action of the problem has been demonstrated, the teacher should lead the following discussion:

"How many groups of cats did we have when we started the problem?" The students should respond, "Two." If they do not, the teacher should replace the two groups of cats by placing five under the tree and three on the doorstep. Next, the teacher should gently guide the children to see that there are two groups of cats. This knowledge must be firmly planted in the mind of the student before proceeding with the problem solution.

The teacher should ask the students to state what the cats did in the problem. If they do not immediately respond, the teacher could say, "The dog barked and the cats jumped on the fence." Encourage students to model this action on their pages using the plastic cats.

The teacher's next comment might be, "With two or more groups, you can either *combine* or *compare* the groups. Which one did we do with the cats?" The students should realize that the action of *combine* occurred.

Finally, the teacher asks, "How many cats are on the fence now?" Through counting or simple addition, the students should be led to respond, "Eight."

When the students have completed the problem with the small plastic models of cats, the teacher should repeat the process using pictures of cats. After discussing the problem using the pictorial level of representation, the students should be instructed to remove the pictures and use "X" marks to designate the cats. Not only does this exercise provide drill and practice for reinforcement of the skill, but it assures the teacher of an opportunity to track the student's movement from concrete to abstract reasoning.

The next step, which might not be taken until the students have fully mastered the basic mathematical understanding, would be writing and solving a number sentence to represent the operation of addition: $5 + 3 = \underline{8}$.

The teacher should repeat the procedures outlined above with the following two-addend problems. It would be important to replicate the manipulation of the cats with each new situation. The following are problem samples you might choose to illustrate:

$$7 + 1 = 8$$

$$6 + 2 = 8$$

$$4 + 4 = 8$$

$$0 + 8 = 8$$

$$1 + 7 = 8$$

$$2 + 6 = 8$$

$$3 + 5 = 8$$

$$8 + 0 = 8$$

Each of these problem examples provide an opportunity to mention the commutative property of addition. Multiple addend problems should also be encouraged as they would afford an opportunity to demonstrate the associative property of addition.

The final step is to ask students to use the writing process to generate new problems. Using a small index card, each child is asked to write a problem story about the cats. After the draft written problems are complete, the students should self

evaluate each one. Next, the problems should be exchanged with those other students in the room. If there are deficiencies, the author- student is informed. The author then revises if necessary and publishes in the best possible form the written word problems.

The important features of this TAPS procedure are teacher modeling; manipulation and discussion of problem elements; identification of the number of groups and the choices of action; understanding the assumed movement, action, or changes; and isolating and comprehending the question of the written problem.

Summary

This text is designed to acquaint the reader with the process of teaching and thinking which is associated with **Talton's Thinking Approach to Problem Solving**. Teachers may wish to follow these guidelines to create their own problem solving materials utilizing local situations and manipulative objects. This approach is commercially available for those who choose to purchase prepackaged materials. (Action Competencies, 1987)

The issue of the thinking process, of information reception, and expression styles have been considered in this chapter with examples of the problem solving process as it is impacted by the student's communication abilities. A discussion of the impact of language, both oral and listening, on the problem solving skills of students has been included. Attention has been given to speech features which might create a barrier to the student's comprehension of the message of a written problem due to linguistic phenomena.

The writing process, as it is related to student written problems has been discussed. Through manipulation of objects, students will quickly insist that they can create their own problem statements. Stumping one another will become somewhat of a game in the classroom thus requiring students to have instruction in the writing process.

The importance of teaching reading comprehension has been considered also. Care should be given to promoting student success in reading through the cueing process associated with manipulation of the objects in their problem display.

Chapter Two contains a thorough discussion of the TAPS model for solving elementary mathematical word problems. The discussion is divided into The TAPS Model, Primary Grades, kindergarten through grade two, where only the operations of addition and subtraction are required and The TAPS Model, Grades Three and Above, which extends the process to include the operations of multiplication and division. Six basic problem types are discussed: three at the primary level, and an additional three above grade two.

Chapter Three contains sample model lessons designed to apply the Thinking Approach to Problem Solving model from kindergarten to grade eight. Future publications will include bulletin boards ideas, folder games, and a variety of opportunities for guided practice. Appendix A provides the Scope and Sequence of problem solving situations which is suggested for use with the TAPS plan. Observation will reveal that once a problem situation has been introduced, it is maintained at each successive grade level.

CHAPTER TWO

TAPS - A THINKING APPROACH TO PROBLEM SOLVING

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TAPS - A THINKING APPROACH TO PROBLEM SOLVING

The Three Basic Actions: Combine, Separate, Compare

More than 95 percent of mathematical reading problems (whether one-step or multi-step) are variations and combinations of three basic actions: *combine*, *separate*, and *compare*. This conclusion was reached by Talton in 1985 and affirmed by Bourgeois in 1987. The first action introduced in elementary mathematics word problems is to *combine* two or more independent groups. The operation introduced in kindergarten and/or first grade to perform this action of combining is addition. Multiplication, or combining equal sets, is introduced usually late in the second grade.

After combining, or joining of sets, has been introduced, children are taught to *separate*, or take away part of a group. This action is presented as the "undoing" of the operation of addition and requires the operation of subtraction. Division is introduced usually in the third grade as the operation to apply when separating a group in such a way that the resulting parts are equal.

When both *combine* and *separate* actions are introduced together, children have their first experiences with decision-making concerning the choice of operation for the solution of word problems. When word problems containing both combine and separate actions are presented together in the same exercise, children must decide whether to add or subtract to solve the problem; that is, they must decide if two or more independent groups are being combined or if one group is being separated into parts.

The third action to be introduced in word problems is *compare to find a difference between two or more groups*. This concept begins an important change in the solving of word problems because, until the introduction of *comparison* word problems, students have been instructed to *combine* when the problem contained two or more groups. Also, with the introduction of comparison problems, students find another application for the subtraction operation. No longer is subtraction used *only* to separate a group into parts. Now, the operation of subtraction is used to *compare* groups to find differences as well as to *separate* a group into parts. In the middle grades, compare problem types are expanded to include *compare two or more groups to find a ratio* and the operation of division is applied to state the ratio in its simplest terms.

The Thinking Approach to Problem Solving (TAPS) Model, Primary Grades

In kindergarten and first grade, word problems are presented primarily through pictures due to the limited reading ability of very young children. The example such as the one shown in Figure 1 is typical of the problem solving situations presented at these two grade levels. The TAPS procedure stresses the importance of using real objects to model problem solving situations whenever possible, followed by manipulating pictorial representations of the same. Much discussion is emphasized concerning the elements of word problems such as placing 7 small models of dogs on the overhead projector, removing 2 of them, and counting the number that remain to demonstrate the problem solution in Figure 1.

Figure 2 summarizes the cognitive schema provided by the TAPS model for selecting the operation in word problems, grades kindergarten to grade two, where the operations of addition and subtraction are applied.

Figure 1

"Word Problems" at the
Pre-Reading Levels



Figure 2

The TAPS Model for Problem Solving,
Kindergarten to Grade Two

READ the word problem.

THINK:

Display groups using concrete models.

Manipulate objects and discuss groups involved.

Analyze situations presented.

Determine assumed and/or implied action(s).

Consider the question asked.

DECIDE:

Combine 2 or more groups by adding.

Separate 1 group into parts by subtracting.

Compare 2 or more groups to find a difference by subtracting.

WRITE the number sentence.

SOLVE the number sentence.

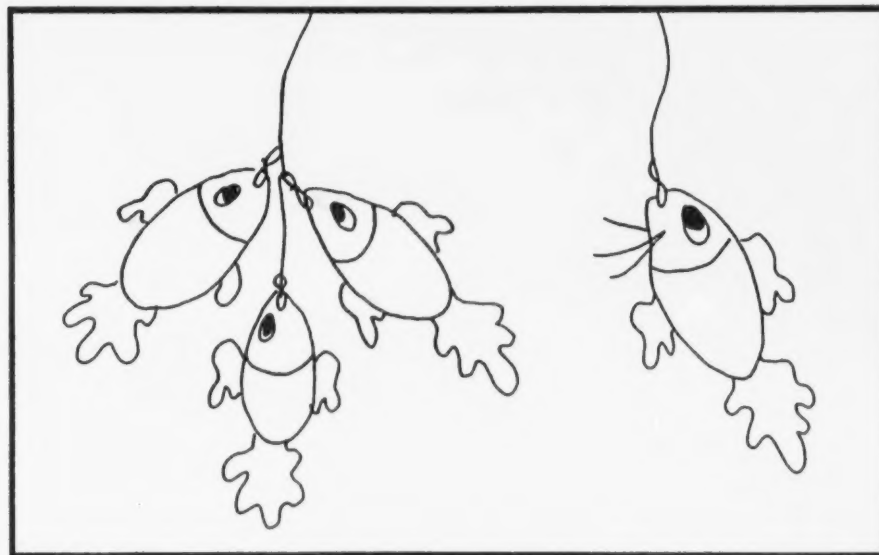
CHECK your answer.

The TAPS model provides teachers an algorithm to teach students how to structure their thinking and focus on *why* a particular arithmetic operation should be used to solve a word problem. Consider the *combine* word problem in Figure 3.

Figure 3

A Problem Example for Combine Two or More Groups
Using Addition

Bill and Ben went fishing last Saturday and caught 3 bass and 1 catfish. How many fish did they catch in all?



After READING the problem, students are led to THINK about the situation posed in the word problem by using manipulative objects to display a model of the group(s) presented in the stated problem. Children are encouraged to manipulate the objects and to analyze the situation presented in the word problem. Often there is an assumed or implied action

which occurred and resulted in the question of the problem being asked. In this example, we assume that Bill and Ben put their fish together into one group. Next, we must very carefully consider the question "How many fish did they catch in all?" This is the place where many children experience difficulty with word problems asking "Do I add or subtract here?" The next step in TAPS, DECIDE, will help them make that decision.

Students have modeled the number of groups in the problem with concrete objects and should see that there are two groups of fish, one group of three bass and one group of one catfish. The question of the problem will tell what action is being taken upon the groups. "How many in all?" indicates that the groups must be *combined*. Instruct students in the primary grades of kindergarten through grade two, to *combine* by adding.

The final steps require the student to translate the action upon the group or groups into a number sentence, WRITE " $3 + 1 = \underline{\quad}$ ", SOLVE by performing the arithmetic computation, $3 + 1 = 4$, and CHECK for reasonableness and accuracy of the answer.

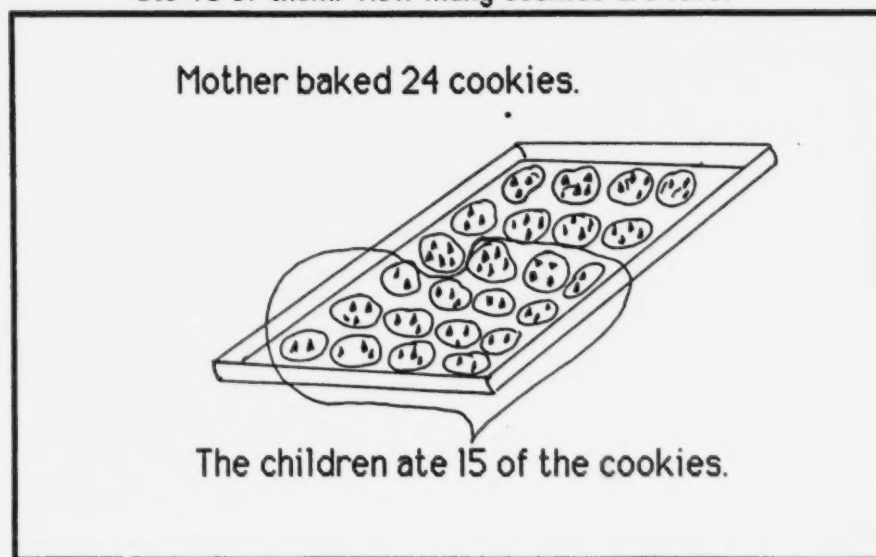
The following example relates to the second problem type found in the primary grades, separate a group into parts, and is illustrated in Figure 4. When the problem has been read, the teacher should lead students to THINK about the situation posed, display concrete models of the 24 cookies that mother baked, and ask students to analyze the group in light of the information provided in the problem. "Is there one group of 24 cookies and a second group of 15 cookies?" Talking about the numbers shows the children that mother baked only one group of 24 cookies and the children ate 15 of them. In a problem such as this, students must assume that the children ate cookies baked by mother; that is, they did not reach up into the kitchen

cabinet and take 15 cookies out of a package bought at a store. Carefully consider the question "How many are left?" This must mean "take away" 15 cookies from the group of 24 and determine how many cookies were *not* eaten.

Figure 4

A Problem Example for Separate One Group into Parts
Using Subtraction

Mother baked 24 chocolate chip cookies. The children ate 15 of them. How many cookies are left?



In the DECIDE step, students must determine what action is being taken upon the groups in the problem. The problem states that one group of 24 cookies was baked and that 15 were separated away. The TAPS model teaches *separate a group into parts* by subtraction in these primary

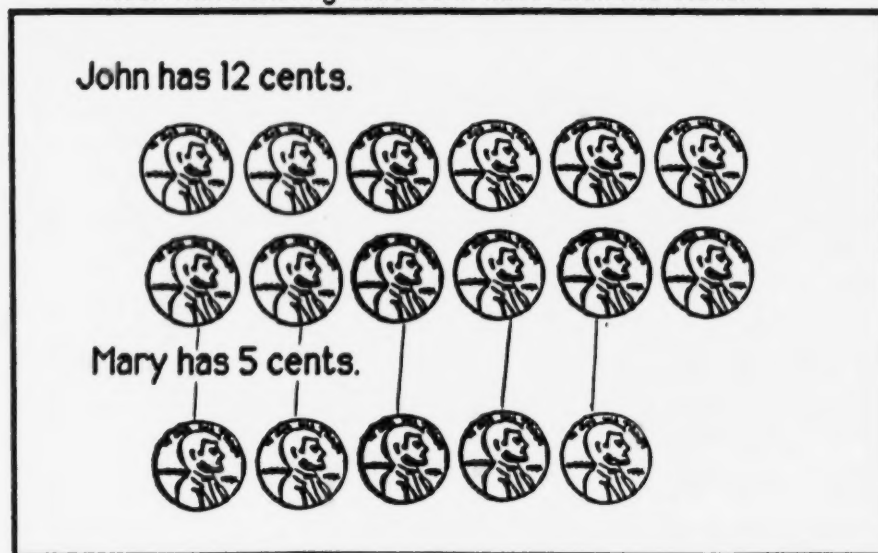
grade levels. WRITE the number sentence " $24 - 15 = \underline{\quad}$ ", SOLVE the sentence, $24 - 15 = 9$, and CHECK for computation errors and reasonableness of the answer. Nine of the 24 cookies were left after the children ate 15.

The third problem type introduced in the primary grades of kindergarten through grade two, is *compare to find a difference between two or more groups*. Figure 5 illustrates an example of this **compare** action and is suggested for introduction at the second grade.

Figure 5

A Problem Example for Compare Two or More Groups to
Find a Difference Using Subtraction

John has 12 cents. His sister Sara has 5 cents. How much more money does John have than his sister?



READ the word problem and use the THINKing strategy outlined by the TAPS procedure. Display the groups using models to represent the 12 cents

that John had and the 5 cents that Sara had. Manipulate the objects and analyze the situation presented so that students will understand that there are two separate groups of money in this problem: John has one amount, 12 cents, and his sister has a second amount, 5 cents.

Using the models of the groups, the student will see that there are two independent groups in the problem. Instruct students that when a word problem has two or more groups, they must DECIDE between *combining* the groups to find "how many in all" or *comparing* the groups to "find a difference" between them. The question asked in the problem posed is "How much more money does John have than Sara?" Instruct students that this problem requires them to *compare to find a difference* between the two groups. Matching the groups in one-to-one correspondence will show that John has more money than Sara. This problem type introduces a new application for the operation of subtraction. Instead of subtracting to separate or "take away" part of a group, students now use subtraction also to *find a difference between groups*. WRITE the number sentence " $12 - 5 = \underline{\quad}$ ", SOLVE the sentence, $12 - 5 = 7$, and CHECK the computation for accuracy. Subtraction reveals that John has 7 cents more than his sister Sara.

Chapter Three of this manuscript provides detailed sample lessons using the TAPS model appropriate from kindergarten to grade eight. In kindergarten, first, and second grades only the operations of addition and subtraction are considered. Each model lesson contains a teaching transparency and an illustrated student Situation Page which directs student thinking from the concrete level, through the pictorial, ending with the symbolic number sentence required for problem solution.

accompanying Teacher's Guide states the lesson objective, prerequisite skill(s), teaching guidelines, teaching procedures, suggestions for student generated problems, and additional practice activities. The concrete objects referenced in the lessons as manipulatives, such as small fish, bears, ants, etc., are available commercially, and often may be found in local toy or department stores.

The TAPS Model, Grades Three and Above

By the third grade, the TAPS model expands to include the operations of multiplication and division (see Figure 6). Multiplication is introduced as another means of combining to find a total. In problem solving applications, multiplication is used to join sets that are *equal* in numerical value. Division is introduced soon after multiplication and is taught as a means of separating one group into *equal* parts. With both the operations of multiplication and division, *equal* is a key word to the application of the operations. Multiplication is used only to *combine* equal groups and the operation of division is used only to *separate* one group into equal parts. Word problem applications of multiplication and division add three additional problem types that are practiced in the elementary grades: (1) *combine equal groups by multiplying*, (2) *separate one group into equal parts by dividing*, and (3) *compare two or more groups to find a ratio and divide* to state the ratio in simplest terms. When these three problem types are added to the three types presented in the primary grades, teachers and students will have a cognitive schema with which to structure their thinking and concentrate on *why* a particular operation is appropriate in a given problem solving situation. These problems represent the six basic problem types identified in Talton's TAPS.

Figure 6

The TAPS Problem Solving Model,
Grades Three to Eight

READ the word problem.

THINK:

Display groups using concrete models.

Manipulate objects and discuss groups involved.

Analyze situations presented.

Determine assumed and/or implied action(s).

Consider the question asked.

DECIDE:

Combine 2 or more groups by addition.

Combine 2 or more *equal* groups by multiplication.

Separate 1 group into parts by subtraction.

Separate 1 group into *equal* parts by division.

Compare 2 or more groups to find a difference by subtraction.

Compare 2 or more groups to find a *ratio* by division.

WRITE the number sentence.

SOLVE the number sentence.

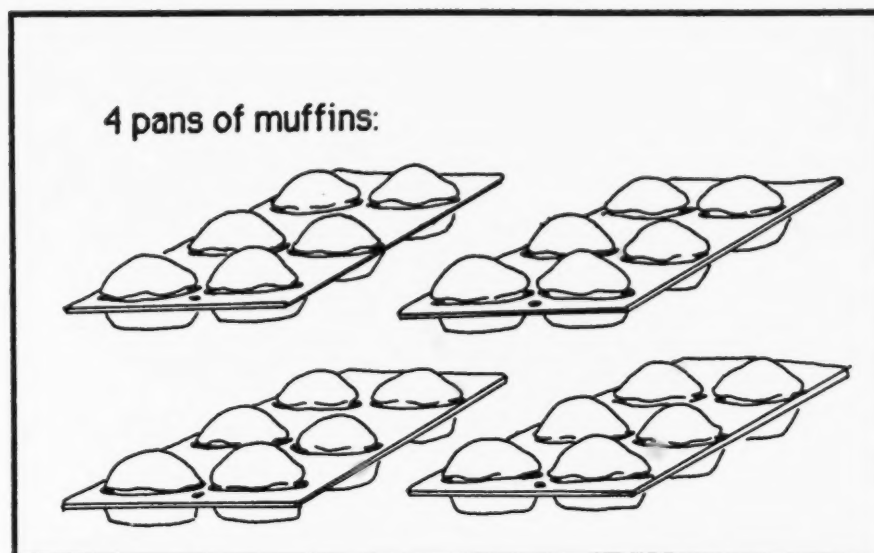
CHECK your answer.

Consider the following three word problem examples which briefly summarize the additional problem types introduced in the intermediate grades. Figure 7 is an example of *combine equal groups using multiplication*.

Figure 7

A Problem Example of Combine Equal Groups
Using Multiplication

Molly baked 4 pans of muffins. Each pan held 6 muffins.
How many muffins did she bake in all?



READ the word problem to determine the mathematical situation presented in verbal form. THINK about the situation presented as manipulative pieces are used to display the groups. "Are there two groups, a group of 4 pans and a group of 6 muffins?" Analyzing the situation displayed with concrete objects will allow students to understand that a

pan containing 6 muffins was baked 4 times. The assumption must be made that each time the pan was placed in the oven, 6 muffins were baked in the pan. Consider the question asked, "How many did she bake in all?" and determine that all of the muffins must be *combined* into one group to find the total.

The DECIDE step involves the critical choice of operations. Since the 4 groups to be *combined* all contain the same number of muffins, 6, multiplication can be applied to find the answer. Now WRITE the number sentence, " $4 \times 6 = \underline{\hspace{2cm}}$ ", SOLVE the number sentence, $4 \times 6 = 24$, and CHECK the answer for accuracy. Molly baked a total of 24 muffins.

Soon after the introduction of multiplication, division is taught. Again, the TAPS model provides a way to reason when that operation would be the appropriate solution strategy to solve a word problem. Consider the problem example in Figure 8.

READ the word problem, THINK about the groups of 35 eggs and 5 baskets, and display them with concrete objects. Manipulate the objects and carefully analyze the situation as presented in the stated problem. The concrete objects will enable students to determine that there is one group of 35 eggs that must be *separated* between the 5 baskets.

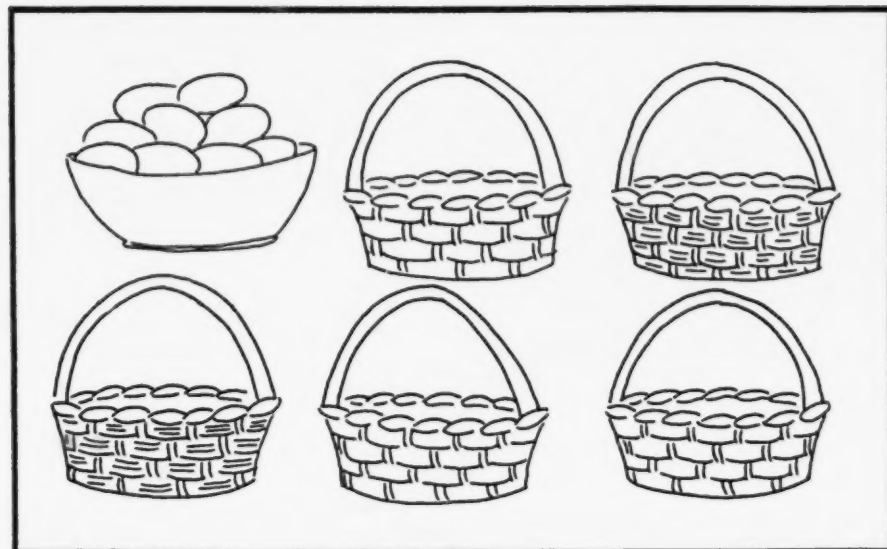
The assumption must be made that the same number of eggs is placed in each basket when the question "How many eggs were in each basket?" is considered even though the problem did not state that specifically. Students will DECIDE to divide as they determine that the problem contains one group that must be *separated into equal parts*. Only when one group is to be separated in such a way that all parts are *equal* should division be used to *separate the group into parts*. WRITE the number

sentence," $35 \div 5 = \underline{\hspace{1cm}}$ ", SOLVE the sentence, $35 \div 5 = 7$ and CHECK the answer for accuracy. Seven eggs will be placed in each basket.

Figure 8

A Problem Example for Separate One Group into Equal Parts
Using Division

Thirty-five eggs which had been dyed for the annual egg hunt were placed in each of 5 baskets. How many eggs were in each one?

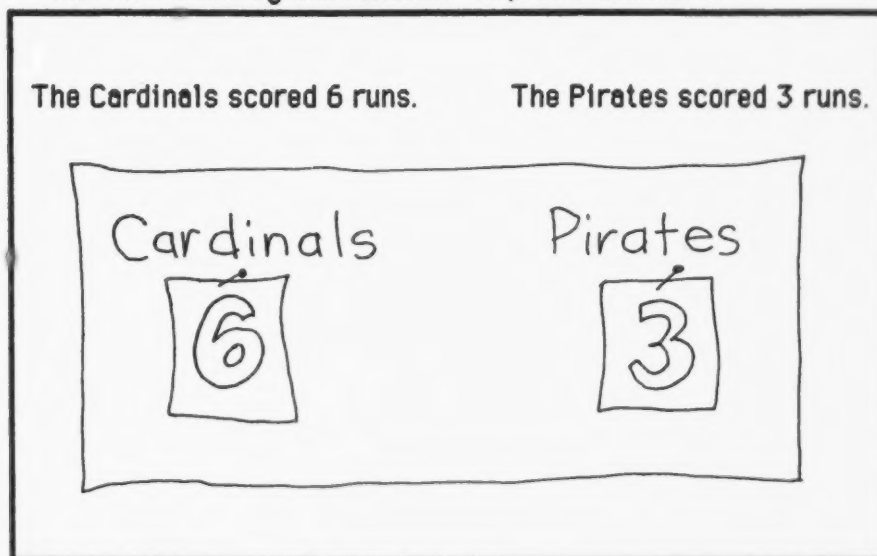


The final type of routine word problem, usually presented by the end of the fifth grade, is a special comparison known as a **ratio**. In these problem types, if both numbers in the ratio have a factor in common, division is applied to simplify that ratio. Consider the example in Figure 9.

Figure 9

A Problem Example for Compare Two or More Groups
to Find a Ratio Using Division to Simplify

The Cardinals scored 6 runs and the Pirates scored 3 runs.
What is the comparison of runs made by the Cardinals to
the runs made by the Pirates? Express in a ratio.



READ the word problem. THINK about the problem situation as concrete objects are used to display and manipulate the two groups. Analyze the groups, 6 runs and 3 runs, before considering the question posed. The problem asked that the Cardinal's score of 6 runs be *compared* to the Pirate's score of 3 runs. WRITE the comparison as 6 to 3, or 6 : 3. To SOLVE this comparison, simplify the statement by dividing both terms by the common factor, 3. The simplified comparison, called a **ratio** would be 2 to 1, or 2 : 1.

The TAPS model provides students an opportunity to use concrete materials when problem solving situations are introduced and to continue using them until they understand the strategies necessary to solve word

problems. To further develop understanding, students are encouraged to pose problems, manipulate groupings, explore multiple approaches and strategies, and talk with one another as a way of arriving at their own explanations, conjectures, and generalizations about problem solving.

Chapter Three contains sample lessons appropriate to grades three to eight. These problem situations include multiplication and division, as well as addition and subtraction. Each lesson contains a teaching transparency, an illustrated student Situation Page where students explore problem situations beginning with concrete objects, move to pictorial representation, and write the abstract symbolization known as a "number sentence." The Teacher's Guide states the lesson objective, prerequisite skill(s) required, teaching guidelines, procedures for student generated word problems, and additional problem applications. Lesson samples for kindergarten, grades one, two, three, four, and six are routine, one step word problems. This has been deliberately planned to carefully teach each problem type individually before combining problem situations in multi-step problems. Lesson samples for the fifth, seventh, and eighth grades are typical, multi-step problems which can be solved by applying the TAPS model strategy. Again, the lessons have been deliberately and carefully planned to introduce the various problem situations one at a time and then in combination.

In summation, this TAPS model will assist teachers to provide instruction that will help students make sense out of solving routine mathematics word problems and become powerful mathematical thinkers, a need cited by the California Curriculum Development and Supplemental Materials Commission and summarized by Burns (1987, pp. 50-51). Only by

moving from the concrete level of conceptualization to symbolic reasoning using numerical representation, can students understand the subtleties of word problems and translate that understanding into an effective problem solving strategy. That movement is provided through the application of the TAPS model as it serves to cognitively map and direct higher level thinking skills.

CHAPTER THREE

SAMPLE LESSONS USING THE TAPS PLAN, KINDERGARTEN THROUGH GRADE EIGHT

A Scope and Sequence of TAPS Applications for the Model Lessons

The six problem types included in the TAPS model for problem solving are explained in the Scope and Sequence chart found in Appendix A. Beginning with kindergarten and progressing to grade eight, the problems as they are introduced in this document are:

Kindergarten. The first problem type introduced to young children is *combining groups*. Whether the groups are equal or unequal, we teach children to add. The reference is made to "unequal" groups so that a distinction can be made later for problems where multiplication is appropriate.

First Grade. *Separate a group into parts* is the problem type provided for first grade students. This skill was probably introduced in kindergarten, so the word problem application is appropriate. The operation of subtraction is applied in this separation process.

Second Grade. Second grade students are introduced to the problem setting *compare two or more groups to find a difference*. This application of subtraction is very different from the previous use for subtraction. Whereas students have been subtracting to separate a group into parts, they will now subtract to compare and find differences between groups.

Third Grade. The operation of multiplication is usually presented near the end of the second grade of school. Therefore, students by the third grade should be mature enough to relate to this operation and apply it in problem settings. The distinction is made now that when *two or more equal groups are combined*, we should multiply.

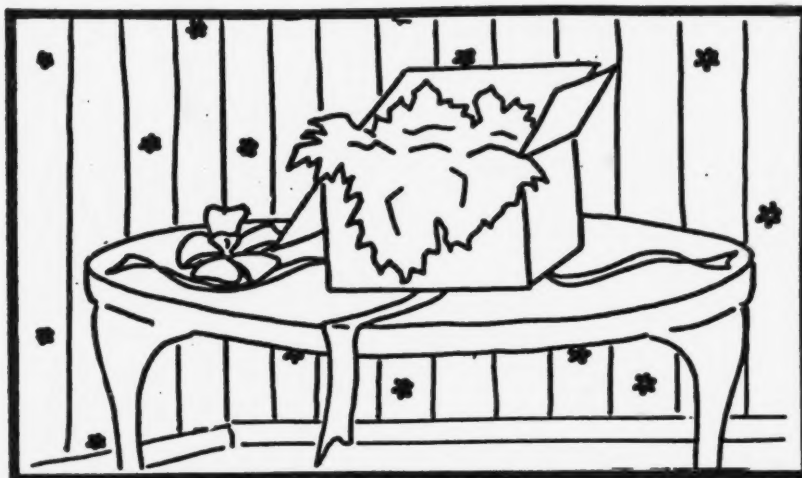
As children are successful with the application of multiplication, introduction of the operation of division becomes appropriate. Students were taught earlier that when *separating a group into parts*, they should subtract. Now, however, an important distinction must be made. After modeling the groups in the problem situation with concrete objects, the TAPS plan teaches students that when the separation process requires the resulting parts to be equal, they should divide.

Fourth and Fifth Grade. No new problem types are introduced in the TAPS model at these two grade levels. Much guided practice should be provided to assure a thorough understanding of the previous five problem types. Problems should be provided in mixed practice. That is, combine situations within the same problem. Traditionally, these problems have been referred to as "multi-step" problems. In the TAPS plan, multi-step problems are introduced as early as the second grade, or whenever students have developed a foundation of the combine and separate actions. Only when this foundation has been provided, can students really understand multi-step problem problems.

Sixth Grade. By the sixth grade, if the preceeding five problem types have been thoroughly presented and practiced, students should have had much experience in a variety of problem solving situations. They should now be ready for the most difficult problem type typically taught prior to high school mathemtics. *Comparing two or more groups to find a ratio* can be introduced very simply as has been done in the sample lesson provided in this document. This very basic problem situation can be expanded to include sophisticated business and real-world applications similar to those found in the "Additional Practice" portion of the sample lessons.

Student Situation Page

Kindergarten



A	B
C	

Teacher's Guide

Grade Level: Kindergarten

Objective: 1. Each student will develop an understanding of the action combine two or more groups by addition.

Prerequisite Skill: 1. Understanding of addition facts to sum of 8.

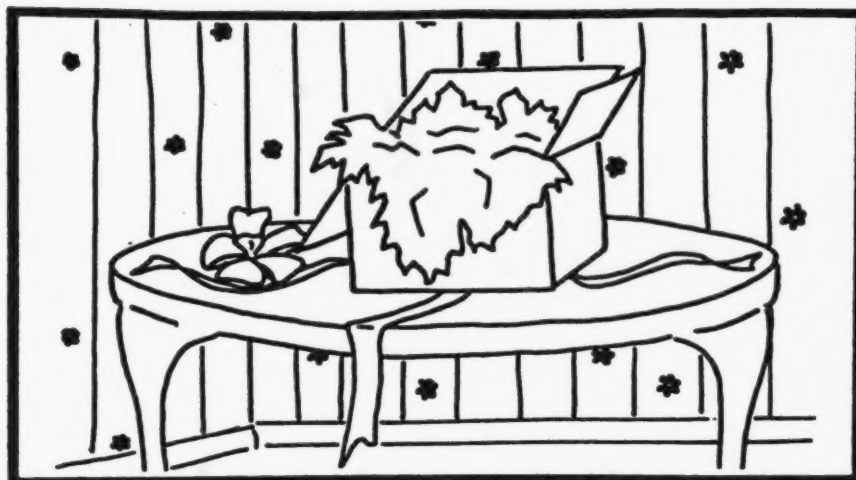
Teacher Guidelines:

1. Model and describe the situation presented on the student's page.
2. Manipulate the display and discuss the various situations and groups involved.
3. Analyze the situation presented.
4. Identify the number of groups.
5. Consider the assumed action(s) and the question of the problem posed.
6. Write the appropriate number sentence to describe the action of the situation.
7. Select a second problem situation to model and repeat steps 1 - 6.
8. Pose other possible problems for consideration and repeat the above procedures.

Teacher Procedures: Situation, Combine

READ: Today was Jerry's birthday. Grandmother would send a gift. She was sure to remember that teddy bears were his favorite toy. When the postman came, a brightly wrapped package arrived. Jerry tore away the paper and ribbon. Was he excited when he saw what was inside. Bears! Bears! There were 2 large bears and 1 small bear inside the package. How many bears in all were inside the package?

1. Place the transparency of the student's Situation Page on the overhead for students to observe. Place 2 large plastic bears and 1 small plastic bear on the table beside the present. Your transparency will be similar to the following:



- (a) Instruct students to display the two groups of bears, 2 large bears and 1 small bear, using concrete manipulatives on the picture of the table in their Situation Pages.
 - (b) Ask students to touch the bears and count aloud as they determine the number of bears that Jerry received for his birthday. Ask "How many large bears did Grandmother send?" The students should respond "Two". Ask "How many small bears did Grandmother send?" Students should respond "one". Repeat the word problem for those students who do not understand the situation presented.
 - (c) Ask "How many different groups of bears did Grandmother send to Jerry?" Encourage students to touch the two large bears and one small bear and answer "Two large bears and one small bear. She sent two groups of bears." Observe students in the classroom to see that all are displaying the problem situation as it was stated.
2. Say "The problem stated that Grandmother sent a birthday present to Jerry. She sent two large teddy bears and one small teddy bear." Point to the two groups of bears on yours transparency as you restate the problem situation. The question asked "How many teddy bears did Grandmother send to Jerry for his birthday." At this point in the problem solution, students must assume that the two groups should be put together into one group and a total number determined.

3. Careful attention will now be given to move students from the concrete level of thought through the pictorial level to the symbolic level of representation using numerals as we combine the groups of bears.

- (a) Instruct students to place the models of the two large teddy bears into Box A on their Situation Pages and the one small teddy bear into Box B. Say "Combine the two groups of bears by moving them both into Box C." Ask "How many bears are there in all?" Say "When a word problem has two or more groups, like the two groups of bears in this problem, we can *combine* the groups. The students' pages should be similar to:

A

B

C

- (b) After moving around the room and determining that students understand the combining action of the two groups of bears using the concrete models, ask them to remove the models and take out the pictures of the bears, 2 large and one small. Instruct students to place the pictures of the 2 large bears into Box A and the picture of 1 small bear into Box B.

Discuss the situation and make sure that the students realize there are 2 groups of bears that must be joined or *combined*. The transparency and their pages should appear as they did in step 3 (a) above.

- (c) When students understand the combining action using the pictorial representation, ask them to remove the pictures and place 2 "X" marks to represent the 2 large bears into Box A and place 1 "X" mark to represent the 1 small bear into Box B. Place the corresponding numerals in each box; that is, place a "2" in Box A and a "1" in Box B. Ask "How many "bears" in all did Jerry's grandmother send to him for his birthday?" The students should be able to answer "Three".

Ask the students to place 3 "X" marks in Box C to represent the 3 bears he received. Instruct them to write the numeral "3" in Box C. Their pages and the transparency should be similar to the following:

A X X = 2	B X = 1
C X X X = 3	

DECIDE 4. Say "To find out how many teddy bears Grandmother sent to Jerry for his birthday, we had to put the two groups together." Say "We have *combined* the two groups of bears and found a total of 3 bears."

WRITE 5. Say "Now write the number sentence in Box C to show what has happened to the two groups of teddy bears in this problem.

(a) Say " *When we combine two or more groups, we add.*"

- (b) Instruct students to write the number sentence that will summarize this combining action in Box C at the bottom of the page. The sentence will be:

$$2 + 1 = \underline{\quad}$$

SOLVE 6. Find the answer to the number sentence:

$$2 + 1 = 3$$

CHECK 7. Check the computation:

$$1 + 2 = 3$$

The computation is correct. Grandmother sent three teddy bears to Jerry for his birthday.

Additional Procedures:

Each of the following problems could be constructed using the manipulative objects and the Situation Layout page in the pupil text. Remember to follow the processes outlined in the Teacher Procedures. Remember to ask "How many groups?" "What is the question we would like to answer?" "What must happen before we can find the answer to the question?" "Will we combine groups?" "What is the number sentence to answer the question?" "Does the answer make sense?" Other problem possibilities to explore are:

- (1) 1 large bear in the box and 2 small bears on the table. How many bears in all?
- (2) 3 small bears on a shelf in the closet and 1 large bear on the bed. How many bears in all?
- (3) 3 small bears in the chair and 0 small bears sitting on a table. How many bears in all?

When appropriate to the skill level of students, explore situations which have three addends such as:

- (1) 1 small bear on the bed, 4 small bears on the chair, and 3 small bears on the floor. How many bears in all?
- (2) 5 large bears ate honey, 2 small bears ate ice cream and 1 small bears ate wild blueberries. How many bears were eating?

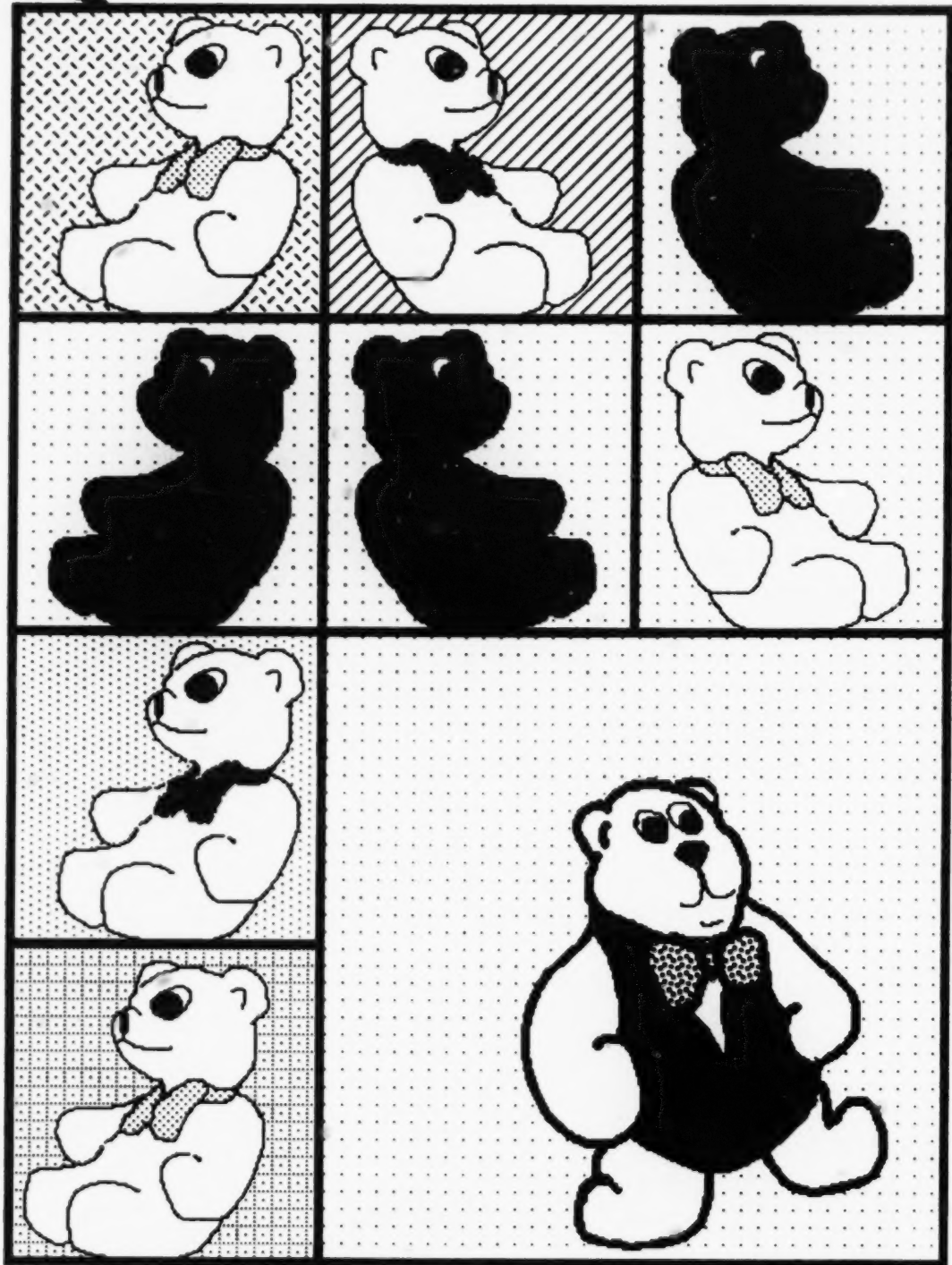
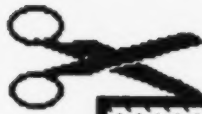
Students should now be encouraged to "write" or compose their own problems. Allow children to work in small groups and make up stories about the bears. Using the language experience approach, have one group tell you the story about their bears as you write down on a large piece of paper what they say. If their story does not include a mathematical situation requiring a solution, offer one of your own to them for their consideration.

Be sure to include questions such as "How many groups of bears are in your story problem?" "Why will we need to join or combine the bears?" Say "Write the number sentence to combine the groups." Draw the conclusion with the children that " *When we combine two or more groups, we add.*"

The following sequence can be used to promote the language experience approach to story problem writing:

- (1) Display several groups of bears on the overhead projector and encourage students to make up a story about the bears.
- (2) Select one of the groups and have the children tell you their story as you write it down. After writing their story problem, ask the children to "read" back to you what they have said. Encourage re-writing to make the story more interesting and to be sure to include a mathematical situation.
- (4) Share the stories among the groups of children. Ultimately, it would be good if these story problems could be shared with other kindergarten classes and/or taken home to be shared with families of the children.

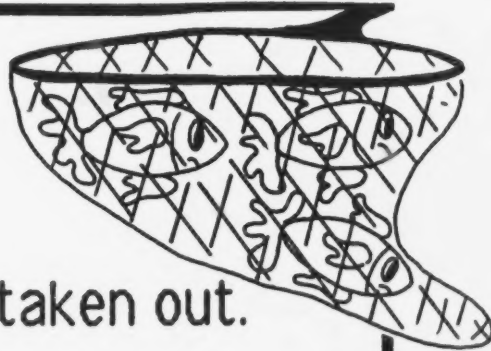
Situation Pictures



Grade 1



4 fish



3 fish are taken out.

How many are left?

Box A

Box B

Box C

Write the number sentence.

Teacher's Guide: Grade One

Situation: Separate One Group into Parts

Objective: Each student will develop an understanding of separate a group into parts by subtraction.

Prerequisite Skill:

1. Counting skills, 1 to 10.
2. Knowledge of addition facts, sums to 10.

Teacher Guidelines:

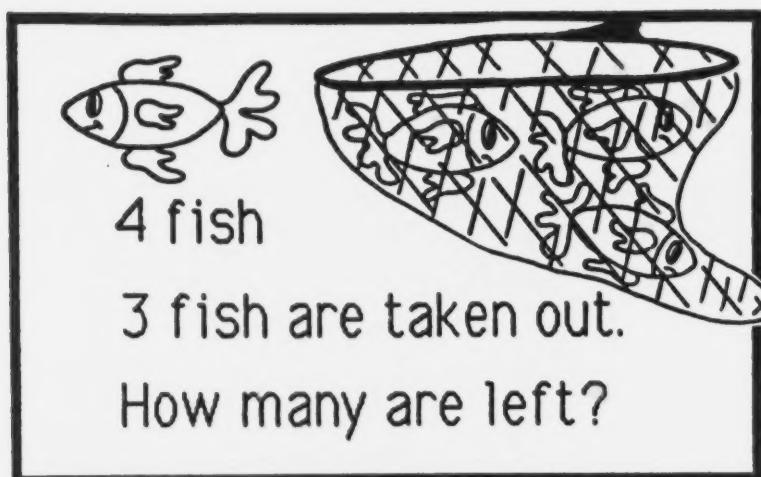
1. Model and describe the situation presented on the student's page
2. Manipulate the display and discuss the various situations and groups involved.
3. Analyze the situation presented.
4. Identify the number of groups.
5. Consider the assumed action(s) and the question of the problem posed.
6. Write the appropriate number sentence to describe the action of the situation.
7. Select a second problem situation to model and repeat steps 1 - 6.
8. Pose other possible problems for consideration and repeat the above procedures.

Teacher Procedures: Situation, Separate a group into parts

READ Four fish were swimming in the pond. A fisherman reached into the pond with a net and dipped out three of the fish. How many were left in the pond?

THINK

1. Use the enclosed transparency of the student's Situation Page and place manipulative objects to represent the 4 fish in the pond. Discuss the situation presented in the problem. Your transparency should be similar to the following:

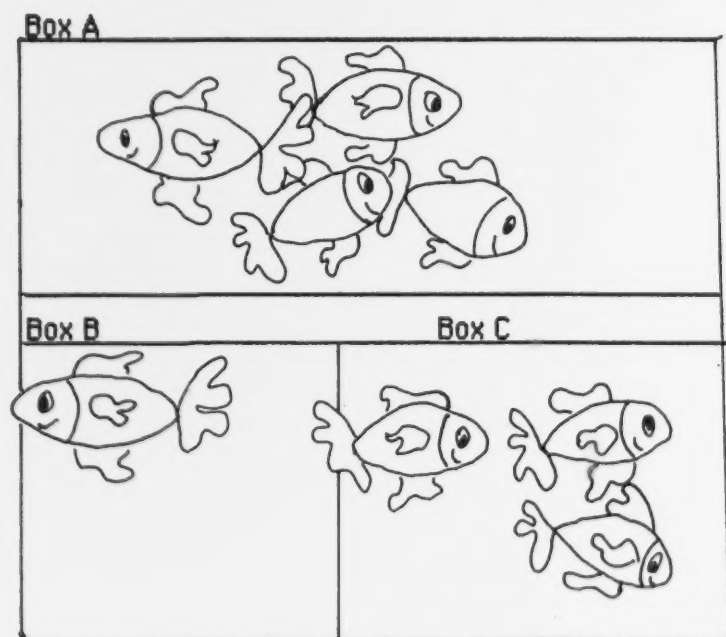


- (a) Instruct students to model the group of fish using concrete objects on their Situation Pages.
 - (b) Ask "How many fish do you see?" Have the students touch and count their fish aloud as you move the pieces on the transparency. They should respond "There are four fish in the pond."
 - (c) Ask "Do you see *one* group of 4 fish swimming?" Students should respond "Yes, there is *one* group of four fish swimming in the pond."
2. Say "The problem stated that a fisherman dipped out three of the fish with a net. How many fish were still in the pond?" Move three of the fish off the transparency and ask students to also pick up three fish and move them off of their pages. Discuss the action that has just happened. Ask "What did we do with the three fish?" The students should respond similar to "We took out the fish that were caught by the fisherman." At this point, the students must assume that a part of the group of fish remained in the pond and it is this remaining part that we must consider for the answer.

3. Careful attention will now be given to transfer students from the concrete level of thinking, through the pictorial, to the symbolic level of understanding using numerals.

- (a) Instruct students to move the concrete models of the fish down on their pages to Box A. This represents the total number of fish that were swimming. Ask "How many fish did the fisherman dip out with his net?" Students should respond "Three".

Instruct the students to move three models of fish down to Box B. This represents the number of fish that were separated from the group. Ask "How many fish were left?" Students should see that one fish was left swimming in the pond. Say "Move the remaining fish down to Box C." Their pages and your transparency should be similar to:



- (b) When students understand the action which has take place using the concrete level of representation, ask them to remove the concrete models and take out four pictures of fish from their packets.

Say "Place four pictures of fish in Box A. Move 3 pictures to Box A to represent the number of fish that were removed from the pond. Move the 1 remaining fish to Box C." Ask "How many fish remained in the pond?" Students should see that 1 fish remained in the pond. Their pages and the transparency should be similar to Step 3 (a) above.

- (c) When students understand the action of the problem using the pictorial level of representation, ask them to remove the pictures and place four "X" marks to represent the total number of fish in the pond in Box A. Place three "X" marks in Box B and one "X" mark in Box C. Write the corresponding numerals. Their pages will be similar to:

Box A _____

X X X X = 4

Box B _____ Box C _____

X X X = 3

X = 1

DECIDE

4. Say "We have separated the group of fish into parts. Part of the group was taken out of the water by the fisherman and part of the group stayed in the pond. When we *separate one group into parts*, we subtract"

WRITE

5. Say "Now write the number sentence to show what has happened to the one group of fish in this problem."

(a) Say "We have *separated* one group of fish into parts. When we separate a group into parts, we subtract."

(b) Instruct students to write the number sentence that will summarize this separating action in the space at the bottom of their pages. The sentence will be: $4 - 3 = \underline{\hspace{2cm}}$

SOLVE

6. Find the answer to the number sentence:

$4 - 3 = 1$ One fish was left in the pond after the fisherman dipped out three fish.

CHECK

7. Check the computation in one of these ways:

$$\begin{array}{r} 4 \\ - 1 \\ \hline 3 \end{array} \quad \text{or} \quad \begin{array}{r} 3 \\ + 1 \\ \hline 4 \end{array} \quad \text{or} \quad \begin{array}{r} 1 \\ + 3 \\ \hline 4 \end{array}$$

The computation is correct. There is one fish left in the pond.

Additional Procedures:

Use the overhead projector to pose similar problems using the subtraction facts that children have been taught. Providing practice in a problem solving setting will help make the operation of subtraction meaningful.

Other problems to pose include:

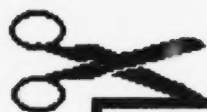
- (1) 8 fish and 3 are caught. How many are left? [5 left]
- (2) 10 fish and 5 are caught. How many are left? [5 left]
- (3) 7 fish and 5 are caught. How many are not caught?
[2 are not caught]
- (4) 9 fish were swimming deep in the water. 2 fish swam
to the top to look for insects. How many stayed deep in
in the water? [7 stayed]

Students should now be encouraged to "write" or compose their own problems. Allow children to work in small groups and make up stories about the fish. Using the language experience approach (explained below) have one group tell you the story about their fish as you write down on a large piece of paper what they say. If their story does not include a mathematical situation requiring a solution, offer one of your own to them for their consideration.

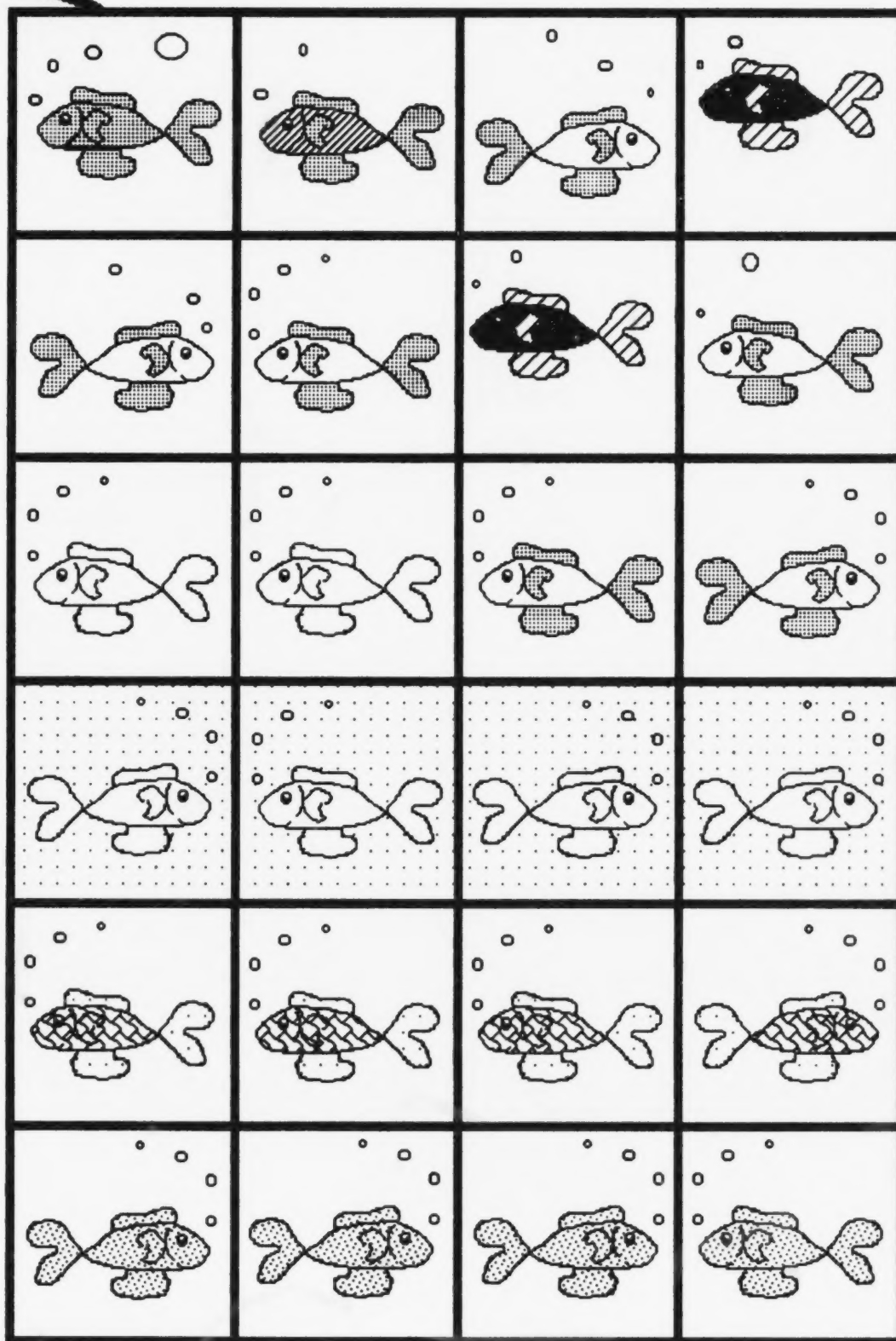
Be sure to include questions such as "How many groups of fish are in your story problem?" "Why will we need to separate the group of fish?" Say "Write the number sentence to separate the fish." Draw the conclusion with the children that " *When we separate one group into parts, we subtract.* "

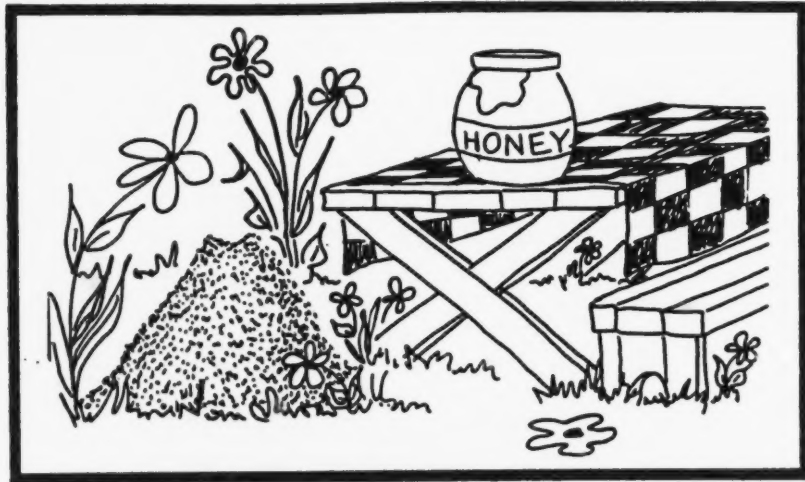
The following sequence can be used to promote the language experience approach to story problem writing:

- (1) Display a group of fish on the overhead projector and encourage students, working in small groups, to make up a story about the fish.
- (2) Select one of the groups of children and have them tell you their story as you write it down. After writing their story problem, ask the children to "read" back to you what they have said. Encourage re-writing to make the story more interesting and to be sure to include a mathematical situation.
- (3) Share the stories among the groups of children. Ultimately, it would be good if these story problems could be shared with other first grade classes and/or taken home to be shared with families of the children.



Situation Pictures





One warm day the Jones family went on a picnic. They took a big jar of honey to eat. They had a good time. When then went home, they left the honey on the picnic table. Nine ants lived under the picnic table in an ant hill. They liked to eat honey, too. They were very hungry. Four of the ants left the ant hill to get honey for the rest of the ants. How many more ants stayed in the ant hill than went to get the honey?

Compare the groups:

Box A

BOX B

Write the number sentence.

Teacher's Guide

Grade Level: 2nd grade

Objective: 1. Each student will develop an understanding of compare to find the difference between two or more groups by subtraction.

Prerequisite Skill: 1. Understanding of addition and subtraction facts to the sum of 9.

Teacher Guidelines:

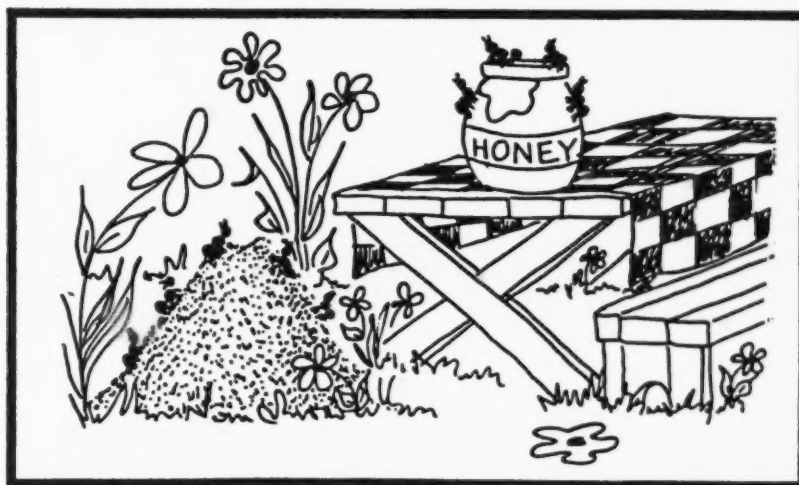
1. Model and describe the situation presented on the student's page.
2. Manipulate the display and discuss the various situations and groups involved.
3. Analyze the situation presented.
4. Identify the number of groups.
5. Consider the assumed action(s) and the question of the problem posed.
6. Write the appropriate number sentence to describe the action of the situation.
7. Select a second problem situation to model and repeat steps 1 - 6.
8. Pose other possible problems for consideration and repeat the above procedures.

Teacher Procedures: Situation. Compare two or more groups to find a difference.

READ: One warm day the Jones family went on a picnic. They took a big jar of honey to eat. They had a good time. When then went home, they left the honey on the picnic table. Nine ants lived under the picnic table in an ant hill. They liked to eat honey, too. They were very hungry. Four of the ants left the ant hill to get honey for the rest of the ants. How many more ants stayed in the ant hill than went to get the honey?

THINK 1. Use the enclosed transparency of the student's Situation Page and place manipulative objects (plastic ants) to represent the nine ants that lived in the ant hill on the transparency. Discuss the problem setting. Say "Four ants went out looking for food. They smelled honey, their favorite food!"

- (a) Instruct the students to display the nine ants on their Situation Pages as you model with the transparency. Then have them to move four of the ants to the honey jar.
- (b) Ask "How many ants went looking for food?" Encourage students to touch the group on the honey jar and count the ants. They should respond "There are four ants looking for food." Your transparency should be similar to the following:



- (c) Ask "How many groups of ants do you see?" Students should respond "There are two groups of ants." If students do not respond "Two", touch each group on the transparency as you say "There is one group of ants that stayed in the ant hill and there is one group of four ants that went looking for honey." Monitor the students to assure that they clearly designate the two groups.

2. The problem stated that four of the ants went looking for food and asked "How many *more* ants stayed in the ant hill than hunted for honey?" At this point in the problem, students have to assume that some of the ants remained in the ant hill although the problem did not state that they did.
3. Careful attention will now be given to move students from the concrete level of thought, through the pictorial, to the symbolic level of understanding using numerals as we make the comparison between the two groups of ants.
 - (a) Instruct students to move the 5 ants that are inside the ant hill down on their pages to Box A. Move the 4 ants that are on the honey jar down on their pages to Box B. Continue to use the transparency of the student's page as you model these actions.

Match the groups in one-to-one-correspondence to show that 5 is 1 greater than 4. To do this, have the students draw a line from each ant in Box A to an ant in Box B. One ant will not be matched. Ask "How many more ants stayed in the ant hill than hunted for honey?" Students should respond "There is one more ant that stayed in the ant hill than hunted for honey." Observe to see that their pages look similar to this configuration:

Compare the groups:

Box A



BOX B



- (b) After determining that students understand the comparing action using the concrete models of the ants, ask them to remove the models and take out the *pictures* of the ants. Instruct the students to place the pictorial representation of the groups of ants on their pages just as they did with the concrete models. The transparency and their pages should appear as they did in step 3 (a) above.
- (c) When students understand the comparing action using the pictorial representation, ask them to remove the pictures and place 5 "X" marks in Box A to represent the ants that stayed in the ant hill and place 4 "X" marks in Box B to represent the ants that went looking for honey. Write the numerical value next to each group of "X" marks. Complete this modeling process at the symbolic level on your transparency as the students write on their pages. Their pages and the transparency should be similar to:

Compare the groups:

Box A	BOX B
X-----X	X-----X
X-----X	X-----X
X-----X	X-----X
X-----X	X-----X
<u>X</u>	<u> </u>
5	4

- DECIDE 4. The problem asked "How many *more* ants stayed in the ant hill than hunted for food?" The question is asking that we compare the two groups to find a difference. Say "When a word problem has *two groups to compare to find a difference*, use subtraction" to solve the problem.

WRITE 5. Write the number sentence on the transparency that will describe the action of comparing that has been demonstrated. It is important that students understand that subtraction is used to compare groups to find a difference as well as to separate a group into parts.

(a) Write the number sentence: $5 - 4 = \underline{\quad}$

(b) Instruct students to write the number sentence in the space at the bottom on their pages.

SOLVE 6. Students should now find the answer to the number sentence. $5 - 4 = 1$. Say "One more ant stayed in the ant hill than hunted for food."

CHECK 7. Have the students evaluate their answers by checking the computation in one of these ways:

$$\begin{array}{r} 5 \\ -1 \\ \hline 4 \end{array}$$

$$\begin{array}{r} 4 \\ +1 \\ \hline 5 \end{array}$$

$$\begin{array}{r} 1 \\ +4 \\ \hline 5 \end{array}$$

The computation is correct.

Additional Practice:

Each of the following problems could be constructed using the manipulative objects and the Situation Layout page in the pupil text. Remember to follow the process outlined in the initial problem. Ask "How many groups are in the problem?" "What is the question that is asked?" Say "When we compare two or more groups to find a difference, use subtraction" Ask "What is the number sentence to find the solution?" Say "Solve the number sentence to find the answer." Ask "Is your answer reasonable and accurate?"

To expand upon the action of *comparing groups to find a difference*, try the following problems using the student Situation Page and the accompanying transparency. Repeat the explanations provided in Step 3 above using the concrete models, pictures, and "X" marks to move students through the three levels of understanding: concrete, pictorial, and symbolic. Ask students to offer other situations involving the comparison of groups of ants.

When students have demonstrated an understanding of comparing two groups to find a difference, try situations that involve comparing more than two groups such as one group in the ant hill, one on the picnic table and another on the honey jar. Although a problem situation may contain more than two groups to be compared, remember that only two groups at a time can be compared. That is why comparisons are considered *binary* operations. Consider the following situation:

Six ants were on the grass outside of the ant hill. *Two* crawled up the leg of the table to reach the honey. *One* ant stayed at home in the ant hill. How many more ants were on the grass than were crawling up the table leg? How many fewer stayed in the ant hill than were in the grass?

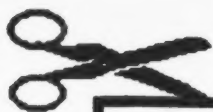
Although this problem situation contains three groups of ants, only two of the groups can be compared at one time. Encourage the students to create similar situations comparing more than two groups of ants.

As students comprehend the problem solving process of comparing two or more groups to find a difference, or for more advanced learners, expand the application to large numbers. The following problem examples use large numbers and could be used to challenge students.

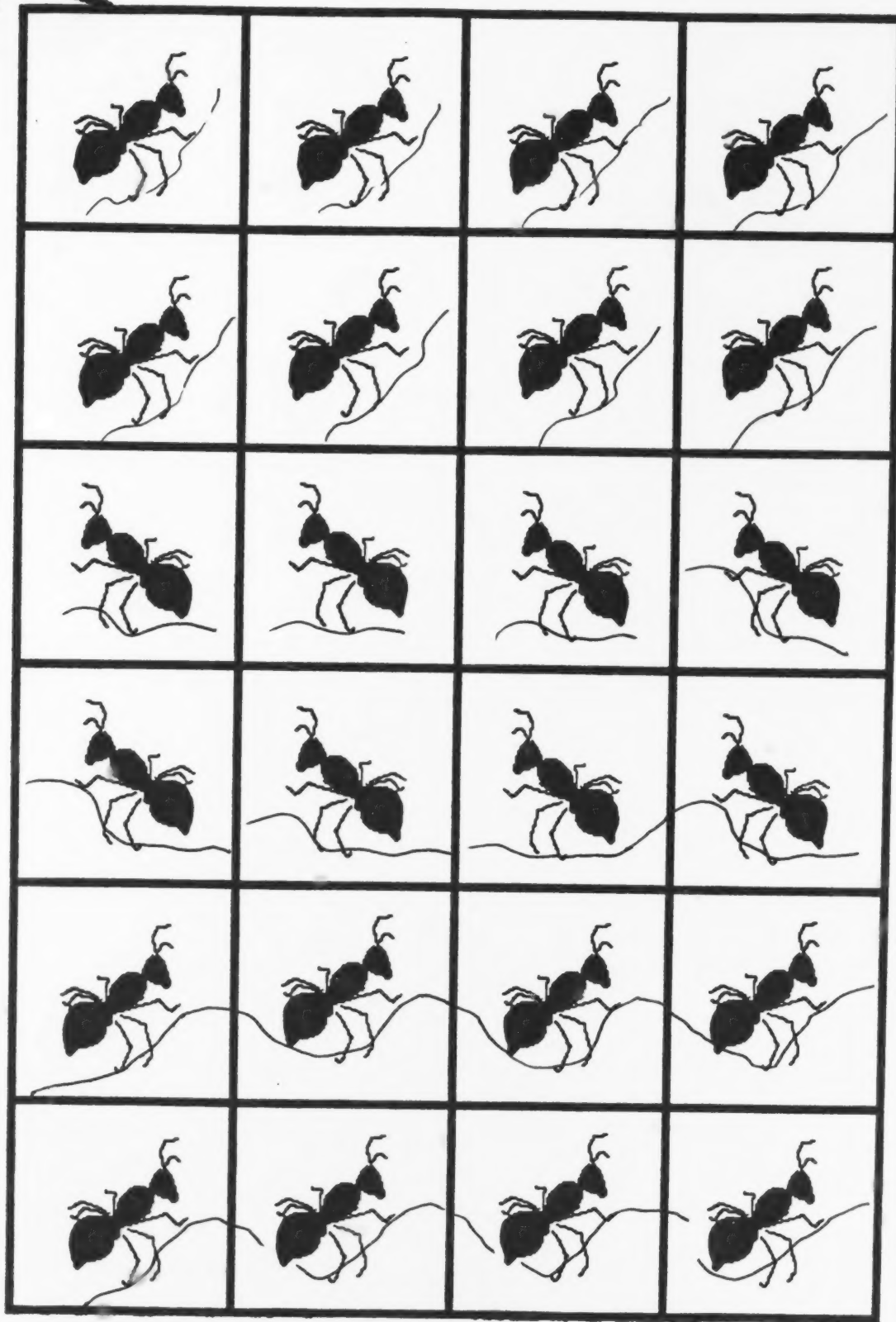
- (1) Ten fire ants built a large mound near the fence post. Eight fire ants built a mound under the apple tree and 11 more decided to build a mound in the open field. How many more ants built a mound in the open field than near the fence post?
- (2) Bill collected information in the library about fire ants for a report in science. He read 2 books, 5 magazine articles, and 4 articles in encyclopedias. How many more magazine articles did he read than books? How many fewer encyclopedia articles did he read than magazine articles?

Students should be encouraged to develop their own written word problems when they have completed several of the problems presented above. The sequence the teacher might choose to follow is

- (1) Display several groups of ants on the overhead projector. Ask students to write a comparing problem about the groups displayed. Remind students that only 2 groups at any one time can be compared.
- (2) When the class has finished drafting their own problems, have them examine them carefully to see that the necessary information has been given to the reader. They should also proofread to determine that the question asked will lead the reader to compare the groups to find a difference.
- (3) Have the students exchange their problems and attempt to solve those written by their peers. Errors which are identified should be corrected by the author.
- (4) Final drafts should be created and the problems published through displaying them on the bulletin board, placing them in a special problem solving box, or using some other strategy for sharing.

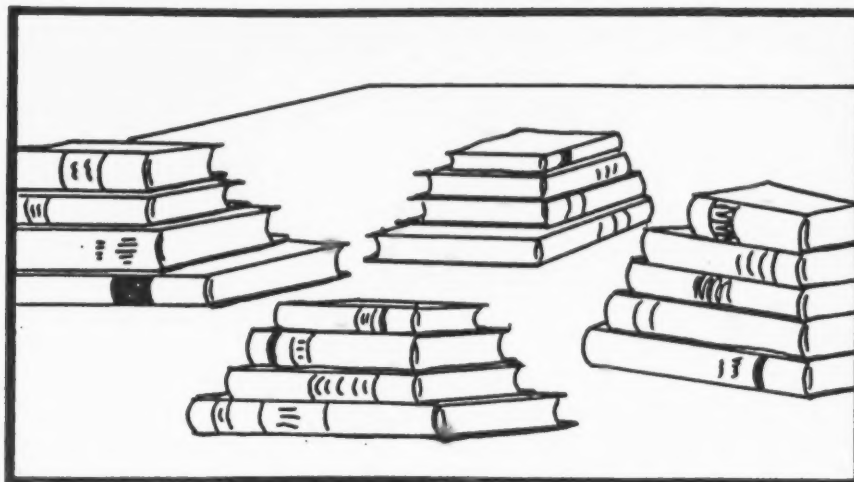


Situation Pictures



Student Situation Page

Grade 3



Juan brought 4 books on farm animals to school for the book collection during the unit on "Farms in America." John added to the collection by bringing 4 books on cows and horses, and Sharon brought 4 books on dairy cows. How many books did they bring together?

Combine equal groups.

Juan's books		John's books		Sharon's books
--------------	--	--------------	--	----------------

Combine the groups:

Write the number sentence.

Teacher's Guide

Grade Level: 3rd grade

Objective: 1. Each student will develop an understanding of combine equal groups by multiplication.

Prerequisite Skill: 1. Understanding of the basic addition and multiplication facts appropriate to needs of problems presented.

Teacher Guidelines:

1. Model and describe the situation presented on student's page.
2. Manipulate the display and discuss the various situations and groups involved.
3. Analyze the situation presented.
4. Identify the number of groups.
5. Consider the assumed action(s) and the question of the problem posed.
6. Write the appropriate number sentence to describe the action of the situation.
7. Select a second problem situation to model and repeat steps 1 - 6.
8. Pose other possible problems for consideration and repeat the above procedures.

Teacher Procedures: Situation, Combine equal groups

READ

Juan brought 4 books on farm animals to school for the book collection during the unit on "Farms in America." John added to the collection by bringing 4 books on cows and horses, and Sharon brought 4 books on dairy cows. How many books did they bring together?

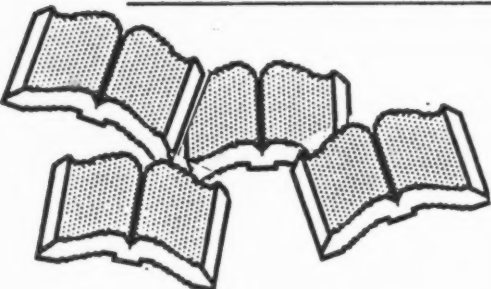
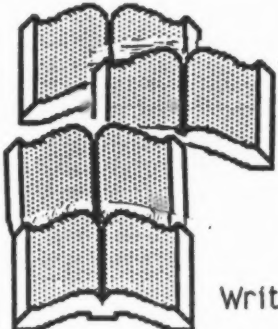
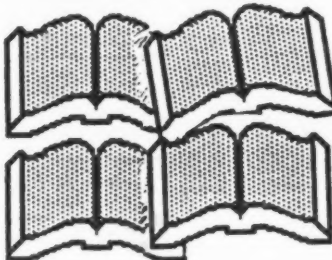
THINK

1. The question asked in the problem was "How many books did they bring together? Use the transparency of the student's Situation Page and place manipulative objects on the transparency to represent the books brought by these three children. Discuss the situation presented in the problem.
 - (a) Instruct students to display the books that each student brought.
 - (b) Ask "How many books did Juan bring?" Students should respond "Four". Ask "How many books did John bring?" Students should respond "Four". Ask "How many books did Sharon bring?" The students should respond "Four". Place four manipulative pieces on the transparency to correspond to the four books brought by each of the three students. Instruct the students to place their manipulative objects to also show four books for each of the three students.
 - (c) Ask "How many groups of books were brought to school?" Students should see that there are three groups of four books. Ask "What do you notice about the groups of books?" Students should respond "The three groups of books are equal. There are four books in *each* group." Encourage students to touch each of the three groups as they answer aloud.
2. The problem asked "How many books did Juan, John, and Sharon bring to the classroom?" The problem does not state that they put their books together, but students must *assume* that they did because it asked "How many did they bring together?" Move the three groups of books into one group on the transparency. Instruct students to also move the three groups together in the section of their Situation Page entitled "Combine the groups." State " *We have combined the three equal groups.* "

3. Careful attention will now be given to move students from the concrete level of thought, through the pictorial, to the symbolic level of representation using numerals as we combine equal groups of books.

- (a) Instruct students to move the concrete models of the books down on their pages. Place the 4 books that Juan brought under his name, the 4 that John brought under his name, and the 4 that Sharon brought under her name. Have the students combine the groups. Your transparency and the students' pages should be similar to:

Combine equal groups.

Juan's books	John's books	Sharon's books
 Combine the groups:	 Write the number sentence. _____	

- (b) After the students demonstrate an understanding of the action of combining equal groups, ask them to remove the concrete models and take out 4 pictures of books for Juan, 4 for John, and 4 for Sharon. Instruct students to place the pictures under the names as they did in Step 3 (a) above. Have the students combine the groups of books on their pages.

- (c) When students understand the combining action of the problem using the pictorial level of representation, ask them to remove the pictures and place "X" marks to represent the number of books that each of the three students brought to school. Have the students combine the "X" marks which represent the books. Their pages and the transparency should be similar to:

Combine equal groups.

Juan's books

John's books

Sharon's books

X X X X = 4

X X X X = 4

X X X X = 4

Combine the groups:

Write the number sentence.

X X X X X X X X X X X X = 12

DECIDE

4. Say "We have *combined equal groups* of books. To find the total number of books, we can multiply.

WRITE

5. Say "Write the number sentence to show what has happened to the three groups of four books."

- (a) Say "We have *combined equal groups* Use the operation of multiplication when combining equal groups.

- (b) Instruct students to write the number sentence that will summarize this combining action in the space at the bottom of their pages. The sentence will be:

$$3 \times 4 = \underline{\quad}$$

SOLVE 6. Find the answer to the number sentence.

$3 \times 4 = 12$. Juan, John, and Sharon brought 12 books to school for the classroom collection.

CHECK 7. Check the computation in one of these ways:

$$\begin{array}{r} 3 \\ \times 4 \\ \hline 12 \end{array} \quad 4 + 4 + 4 = 12, \text{ or } 12 \div 3 = 4, \text{ or } 12 \div 4 = 3$$

The computation is correct. Juan, John, and Sharon together brought 12 books to the class.

Additional Practice:

Encourage students to try other examples which involve the actions of *combine equal groups by multiplication*.

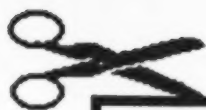
- (1) There were 3 books in the library on the history of England, 3 on the history of Holland, 3 books about France and 3 about Australia. How many books on other countries were in the library?
- (2) Fred is in a book club for stamp collectors. If he received four books during each of the months of January, February, and March of last year, how many books did he receive?

Students should be encouraged to develop their own written word problems when they have completed several of the problems presented above. The sequence the teacher might choose to follow is presented in the following section.

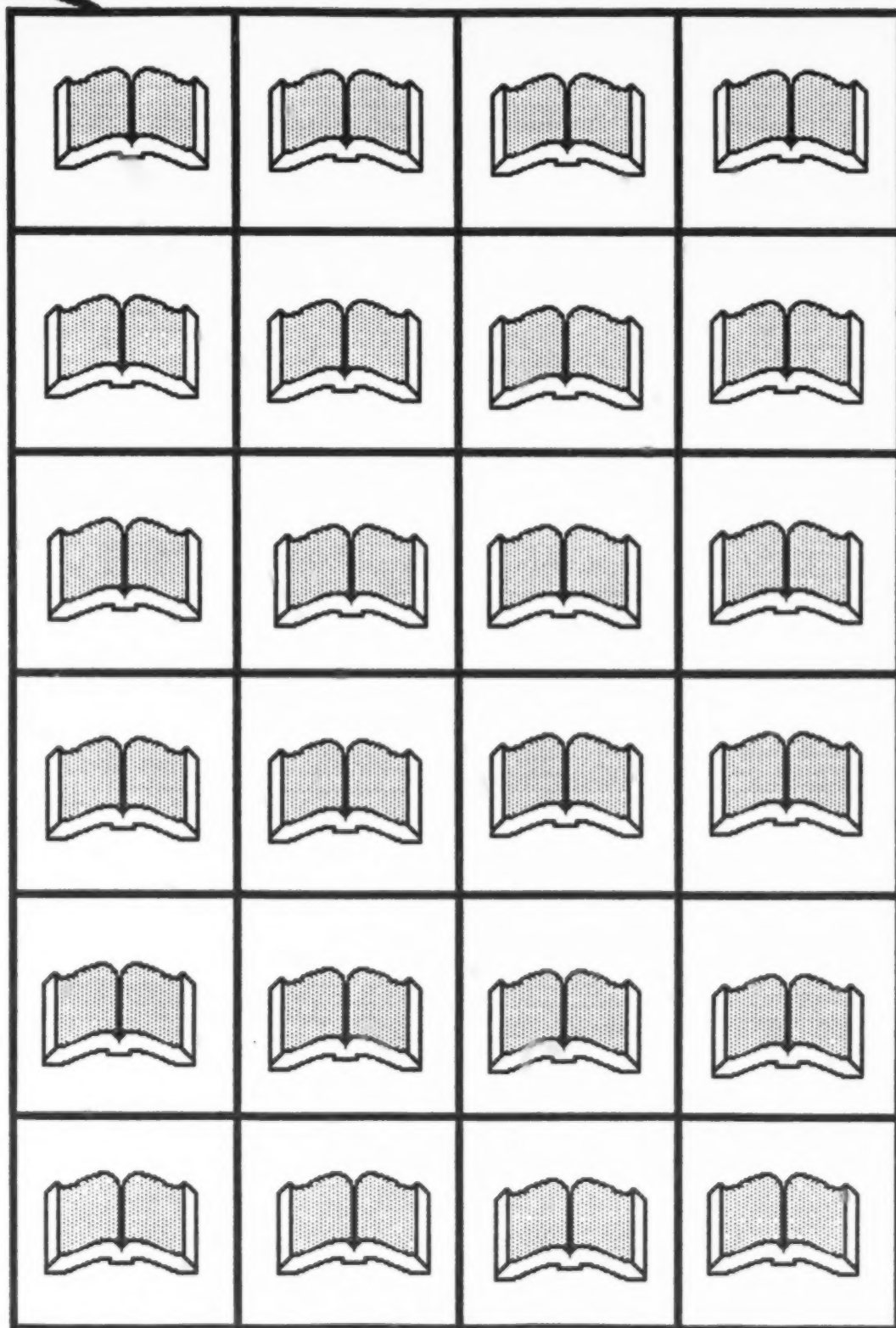
- (1) Display several of the manipulatives of books. Alternatively have the students use actual books that are in the classroom, such as spelling or mathematics books. They should draft copies of problems which require that the reader combine equal groups. For example, one

such problem might be: Shirley's group has 8 readers, Sally's group has 8 readers, too. If Mike's group adds their 8 readers of another kind to the rest of the books in the class, how many books will there be in all?

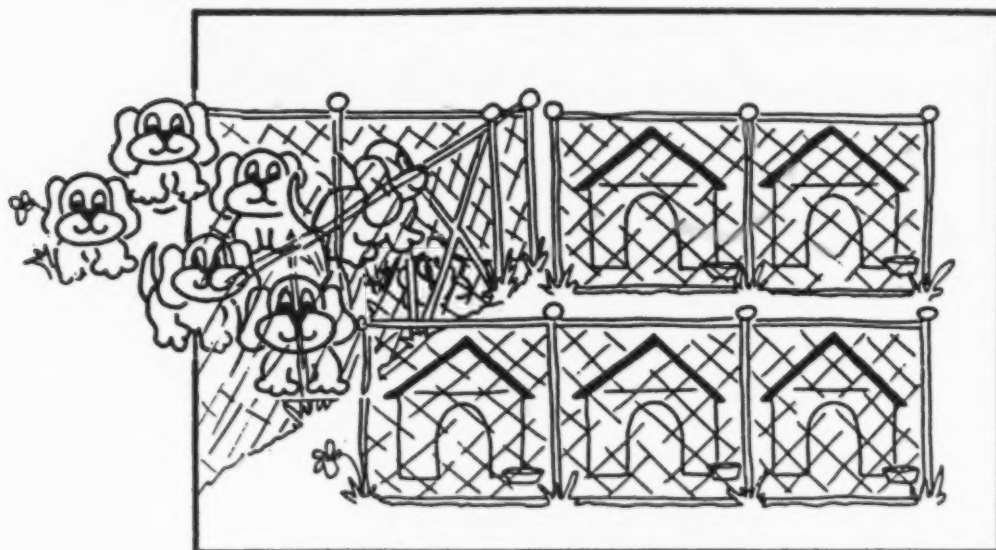
- (2) When the class has finished drafting their problems, have them examine them carefully to see that the necessary information has been given to the reader. They should also proofread to determine that the groups are of equal size. The question of the problem should lead the reader to combine the groups.
- (3) Have the students exchange their problems and attempt to solve those written by their peers. Errors which are identified should be corrected by the author.
- (4) Final Drafts should be created and the problems published through displaying them on the bulletin board, placing them in a special problem solving box, sharing with other third grade classes, or using some other strategy for sharing.



Situation Pictures



Grade 4



At the Gaines Creek Hunt Club, there were fifteen fine hunting dogs that members could use when they went bird hunting. During the day, all of the dogs played together in a large, sunny pen. At night and when it was time to eat, the dogs were separated into five small pens. In this way, each dog received enough food and water and was very comfortable when the weather was cold. How many dogs were in each of the five pens?

<u>Box A</u>	<u>Box B</u>	<u>Box C</u>	<u>Box D</u>	<u>Box E</u>

Write the appropriate number sentence from above.

Teacher's Guide

Grade Level: Grade Four

Objective: 1. Each student will develop an understanding of the concept separate one group into equal parts by dividing.

Teacher Guidelines:

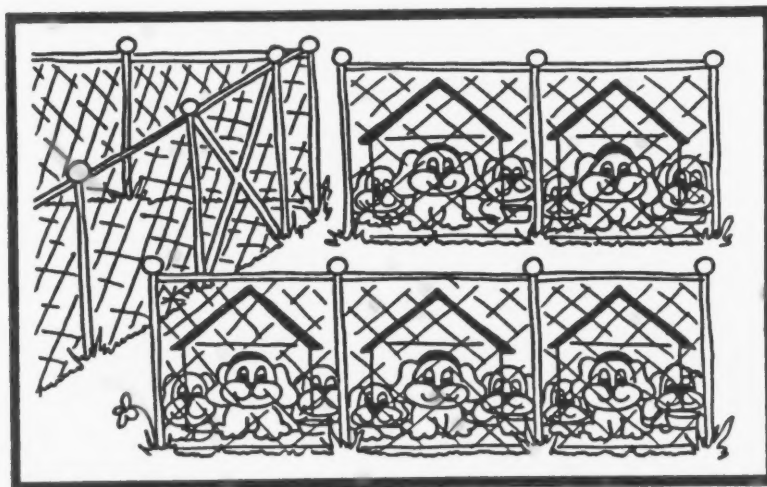
1. Model and describe the situation presented on the student's page.
2. Manipulate the display and discuss the various situations and groups involved.
3. Analyze the situation presented.
4. Identify the number of groups.
5. Consider the assumed action(s) and the question of the problem posed.
6. Write the appropriate number sentence to describe the action of the situation.
7. Select a second problem situation to model and repeat steps 1 - 6.
8. Pose other possible problems for consideration and repeat the above procedures.

Teacher Procedures: Situation, Separate one group into equal parts

READ At the Gaines Creek Hunt Club, there were fifteen fine hunting dogs that members could use when they went bird hunting. During the day, all of the dogs played together in a large, sunny pen. At night and when it was time to eat, the dogs were separated into five small pens. In this way, each dog received enough food and water and was very comfortable when the weather was cold. How many dogs were in each of the five pens?

THINK 1. Use the transparency of the student's Situation Layout page and place 15 dogs inside the fenced yard.

- (a) Instruct students to display concrete models of the groups presented in the problem. The transparency and the students' pages should be similar to:



- (b) Ask "How many dogs were in the large pen at the Gaines Creek Hunt Club? Students should respond "Fifteen".
- (c) Ask "Into how many pens are the dogs separated at night and for feeding?" Students should recall that the problem stated there were five pens where the dogs were kept at night and placed for feeding.
2. Say "The problem stated that the dogs were placed into five small pens to be fed." The students must assume that the dogs were separated equally into the pens.
3. Careful attention will now be given to move the students from the concrete level of thought through the pictorial level and eventually to the symbolic level using numerals as we separate the one group of dogs into equal parts.

- (c) When students understand the separating action of the problem using the pictures, ask them to remove the pictures and place "X" marks in each Box, A - E, to represent the number of dogs in each pen. Place the numeral beside each group of "X" marks to indicate the number of dogs in each pen. This step will help connect the separating action of the problem with the symbolic representation of that number. The transparency and their pages should be similar to:

Box A	Box B	Box C	Box D	Box E
XXX = 3	XXX = 3	XXX = 3	XXX = 3	XXX = 3

- DECIDE 4. Say "We have separated the one group of fifteen dogs into five equal parts. When we *separate one group into equal parts, we can divide.*"

- WRITE 5. Say "Now write the number sentence to show what has happened to the group of fifteen dogs in this problem."

- (a) Say "We have separated one group of fifteen dogs into five equal parts. *When separating one group into equal parts, divide.*"

- (b) Instruct students to write the number sentence that will summarize this separating action in the space at the bottom of their pages. The sentence will be:

$$15 \div 5 = \underline{\quad}$$

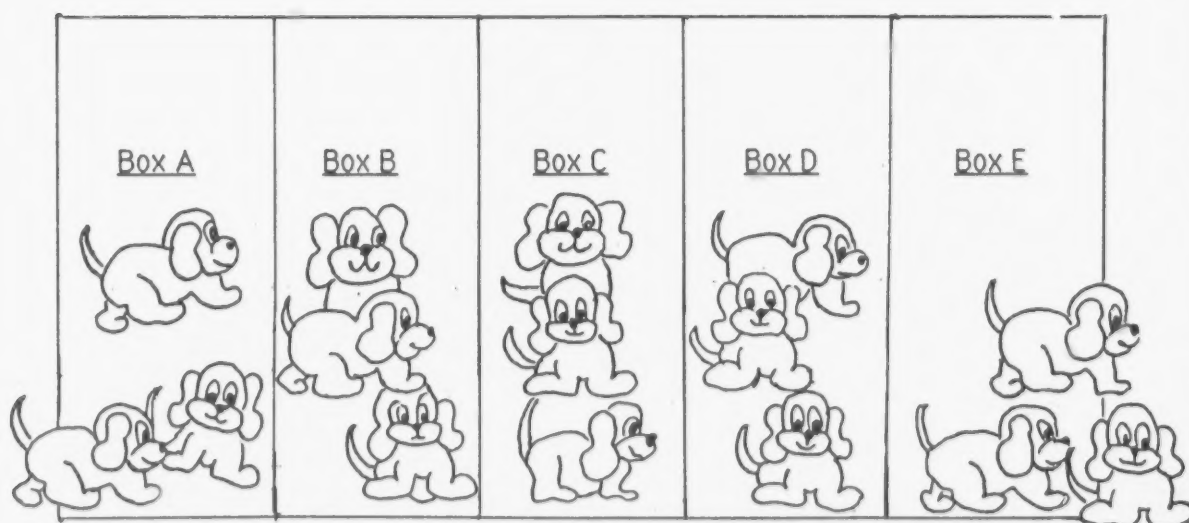
- SOLVE 6. Find the answer to the number sentence. $15 \div 5 = 3$.

- CHECK 7. Check the computation in one of these ways:

$$15 \div 3 = 5, \quad 3 \times 5 = 15, \quad 5 \times 3 = 15.$$

The computation is correct. Three dogs will be in each pen.

- (a) Instruct students to move the 15 dogs down on their pages to Boxes A - E. Distribute the dogs equally among the 5 boxes which represent the small pens. Remind the students that the problem made the assumption that an *equal* number of dogs were placed in each pen. As the problem did not state the number of dogs in each pen, demonstrate to the students the distributive type of division. That is, lay the fifteen dogs out and place the first dog in Box A, the second dog in Box B, the third dog in Box C, etc., until all fifteen dogs have been distributed into the five pens. The transparency and their pages should be similar to:



- (b) After determining that students understand the action of separating one group into equal parts using the concrete models of the dogs, ask them to remove the models and take out 15 pictures of dogs. Instruct the students to separate the pictorial representation of the 15 dogs into Boxes A - E on their pages just as they did with the concrete models. The transparency and their pages will be similar to Step 3 (a) above.

Additional Practice:

Each of the following problem situations could be constructed using the manipulative models and pictures of the dogs, and the Situation Page in the pupil text. Remember to follow the process outlined in the steps 1 - 7 above emphasizing such points as "What is the group that is to be separated?" "Will the group be separated in such a way that the resulting parts are equal?" "Will there be anything left over as a remainder when the separation into equal parts has occurred?" Provide ample practice separating one whole group into various parts. Make the important generalization that "If we want the parts to be *equal* when separating a group, divide." A few examples to explore are:

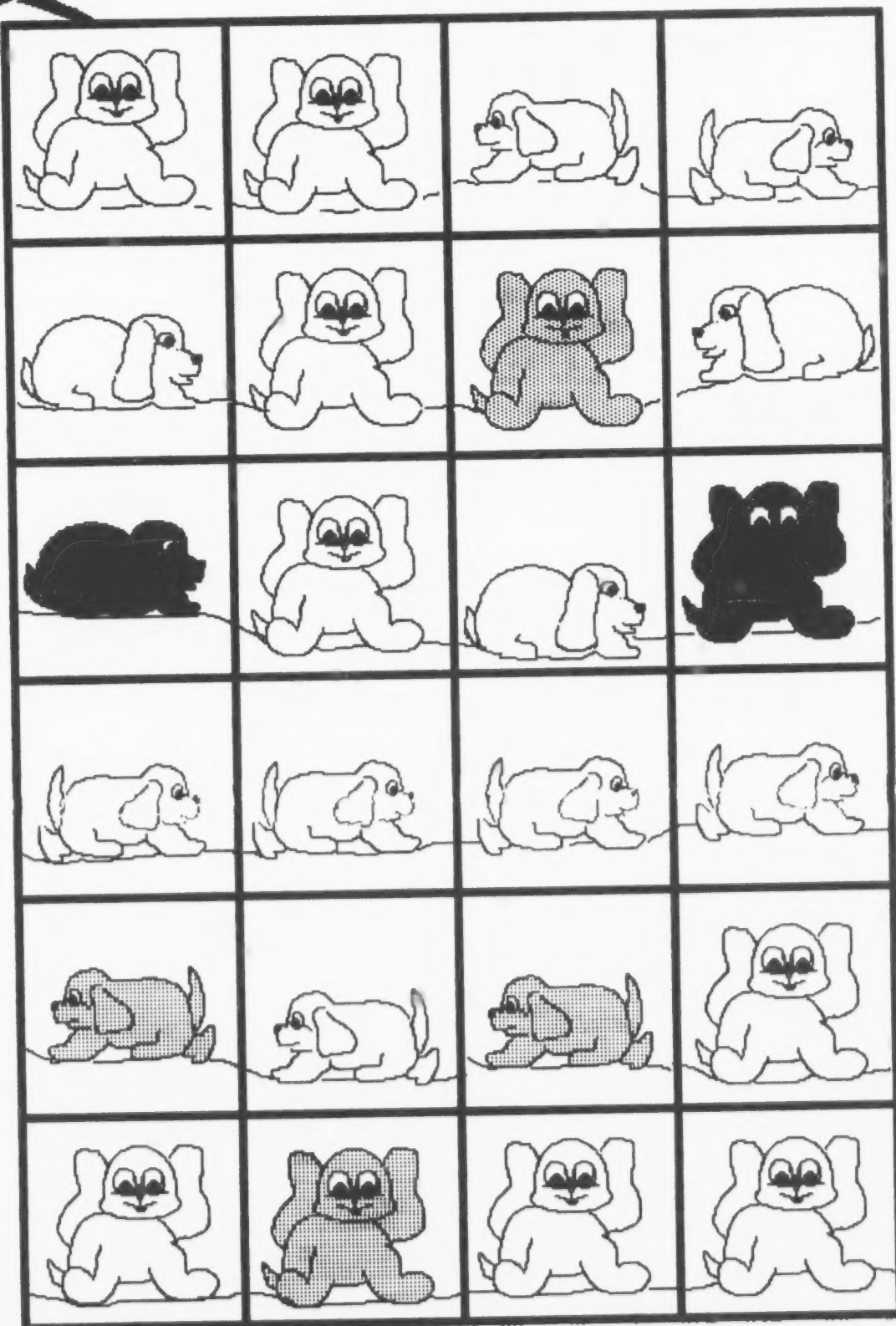
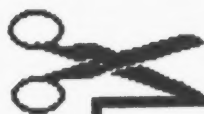
- (1) There were nine dogs at the kennel which needed to be taken to the veterinarian for their shots. Kim had room for three dogs in her little car. How many trips would it take for her to take all of the dogs to get their shots?
- (2) Mark had a part-time job after school working in the kennel taking care of the animals. He especially loved the dogs. He took care of 25 in all. Each day he bathed 4 of the dogs. How many days would it take to get each dog bathed?
- (3) Last week at the state fair, 27 dogs were judged as winners in various categories. If 1st, 2nd, and 3rd place ribbons were awarded, how many categories were judged?

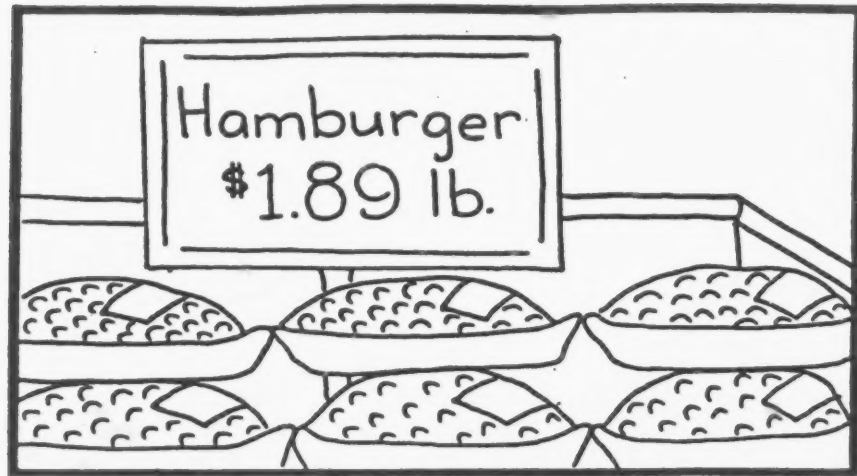
Students should be encouraged to write their own word problems when they have developed an understanding of the action *separating into equal parts*. A sequence the teacher might choose to follow is:

- (1) Display several groups of dogs on the overhead projector. Ask students to write a problem about the dogs in which they are separated into equal parts.

- (2) When the class has finished drafting their own problems, have each student examine his word problem carefully to see that the necessary information has been given to the reader. Students should also proofread to determine if the question asked will lead the reader to separate the group into equal parts. On the pupil *Write Your Own Problems* work pad, the questions have been provided to guide the processing such that each student should be able to compose error free problems.
- (3) Have the students exchange their problems and attempt to solve those written by their peers. Errors which are identified should be corrected by the author.
- (4) Final drafts should be created and the problems published through displaying them on the bulletin board, placing them in a special problem solving box, exchanging with classrooms of the same grade level, or some other strategy for sharing.

Situation Pictures





Stan and Janice were planning their class spring picnic at the beach. Stan bought $2\frac{1}{4}$ lbs of hamburger meat and Janice bought 3 times that amount. How many pounds of hamburger meat did Janice buy? How many pounds of hamburger meat did they buy together? If the meat costs \$1.89 per pound, what is the total cost of the meat?

Situation 1: Combine equal groups.

Amount of meat bought by Stan:

Amount of meat bought by Janice:

Combine the groups and find the total amount bought by Janice.

Write the number sentence: _____

Teacher's Guide

Grade Level: Grade Five

- Objectives:**
1. Each student will maintain his/her understanding of the concept combine equal groups by multiplication.
 2. Each student will maintain his/her understanding of the concept combine unequal groups by addition.

- Prerequisite Skills:**
1. Multiplication of mixed fractions.
 2. Convert mixed fractions to decimal equivalent.

- Teacher Guidelines:**
1. Model and describe the situation presented on student's page.
 2. Manipulate the display and discuss the various situations and groups involved.
 3. Analyze the situation presented.
 4. Identify the number of groups.
 5. Consider the assumed action(s) and the question of the problem posed.
 6. Write the appropriate number sentence to describe the action of the situation.
 7. Select a second problem situation to model and repeat steps 1 - 6.
 8. Pose other possible problems for consideration and repeat the above procedures.

Teacher Procedures: Situation 1. Combine equal groups.

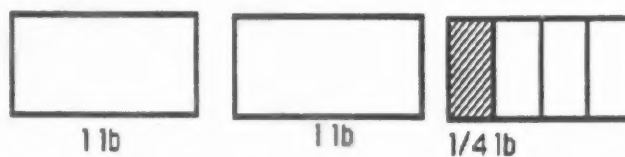
READ

Stan and Janice were planning their class spring picnic at the beach. Stan bought $2\frac{1}{4}$ lbs of hamburger meat and Janice bought 3 times that amount. How many pounds of hamburger meat did Janice buy?

THINK

1. Use the transparency of the student's page and place the models to represent $2\frac{1}{4}$ lbs of hamburger meat bought by Stan.
 - (a) Have the students place the models to represent the hamburger meat that Stan bought on the student Situation Page in the section "Amount of meat that Stan bought."
 - (b) Say "The problem stated that Janice bought *3 times* the amount that Stan bought. Take out 3 sets of models for $2\frac{1}{4}$ lbs of hamburger meat and place the pieces in the section 'Amount of meat than Janice bought'."
2. Say "The problem asked that we find out the amount of meat that Janice bought. We will need to *combine equal groups*"
3. Careful attention will now be given to move the students from the concrete level of thought through the pictorial level and eventually to the symbolic level using numerals as we combine 3 equal groups of hamburger meat that Janice bought.
 - (a) Instruct the students to move the models of the 3 groups of hamburger meat that Janice bought down on their pages to the section "Combine the groups and find the total amount bought by Janice." Say "We must find out how much meat this is in all."
 - (b) When students understand the meaning of the word problem using the concrete models, ask them to remove the models and take out the fraction strips that will represent the hamburger meat at the pictorial level. The amount that Stan bought should be shown similar to the following diagram:

Amount of hamburger meat bought by Stan:

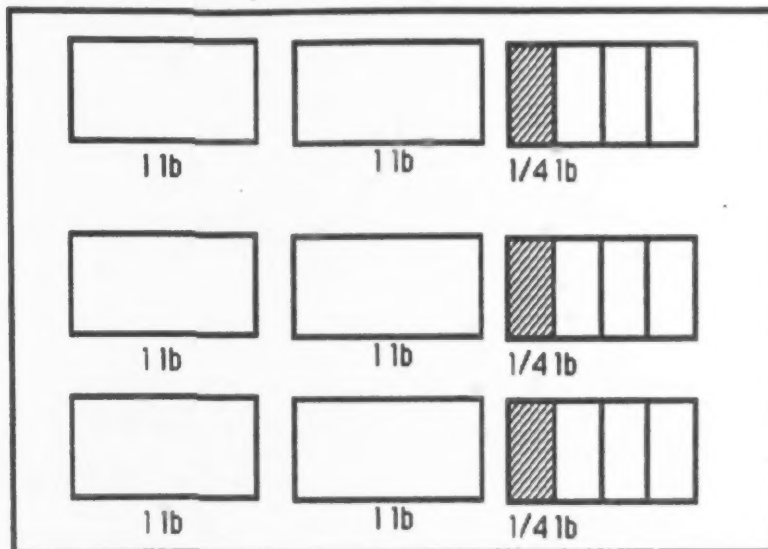


Stan bought 2 $\frac{1}{4}$ lbs of hamburger meat for the picnic.

"The problem stated that "Janice bought 3 times the amount that Stan bought (2 $\frac{1}{4}$ lbs)." The assumption of the problem is that Janice bought the *same amount* that Stan bought the equivalent of *three times*

Instruct students to lay fractions strips out on their Situation Pages to represent 2 and $\frac{1}{4}$ lbs. 3 times. This illustration will represent the amount of hamburger meat that Janice bought. The transparency and the students' pages should look similar to:

Amount of hamburger meat bought by Janice:



- (c) When students understand the combining action required in the problem using the pictorial representation, instruct them to *draw* three groups of fraction strips to represent 2 and $\frac{1}{4}$ lbs of hamburger meat bought by Janice. Observe to see that students understand the relationship in this fraction. They should draw two whole strips and $\frac{1}{4}$ of another strip to represent the amount bought by Stan and draw the illustration *three times* to represent the amount bought by Janice.

The drawings on their Situation Pages and on your transparency should be similar to the illustrations in Step 3 (b) above.

DECIDE

4. Ask "What action is required to determine the amount of meat that Janice bought?" Students should respond "Combining. We are combining 2 $\frac{1}{4}$ lbs of meat 3 times." Using the visual fraction strips should enable students to see that the problem is combining *equal* groups; that is, there is a group of 2 $\frac{1}{4}$ lbs, three times. Multiplication is the choice of operation when combining *equal* groups.

WRITE

5. Say "Write the number sentence to show what has happened to the three groups in this problem."

(a) Say "When combining groups that are *equal*, use the operation of multiplication."

(b) Instruct students to write the number sentence that will summarize the combining action of this problem at the bottom of their pages. The sentence will be $3 \times 2 \frac{1}{4} = \underline{\hspace{2cm}}$

SOLVE

6. Find the answer to the number sentence.

$$3 \times 2 \frac{1}{4} = (3 \times 2) + (3 \times \frac{1}{4})$$

$$= 6 + \frac{3}{4}$$

$$= 6 \frac{3}{4}$$

Janice bought $6 \frac{3}{4}$ lbs of hamburger meat for the picnic.

CHECK

7. Check the computation in one of these ways:

$$2 \frac{1}{4} \times 3 = (2 \times 3) + (\frac{1}{4} \times 3)$$

$$= 6 + \frac{3}{4}$$

$$= 6 \frac{3}{4}$$

$$\text{or } 6 \frac{3}{4} \div 3 = (6 \div 3) + (\frac{3}{4} \div 3)$$

$$= 2 + (\frac{3}{4} \times \frac{1}{3})$$

$$= 2 + \frac{3}{12} \text{ (or } \frac{1}{4})$$

$$= 2 \frac{1}{4}$$

The computation is correct. Janice bought $6 \frac{3}{4}$ lbs of hamburger meat for the school picnic.

Student Situation Page

Situation 2: Combine unequal groups.

Hamburger meat bought by Janice.

Hamburger meat bought by Stan.

Combine the two amounts of hamburger meat.

Write the appropriate number sentence.

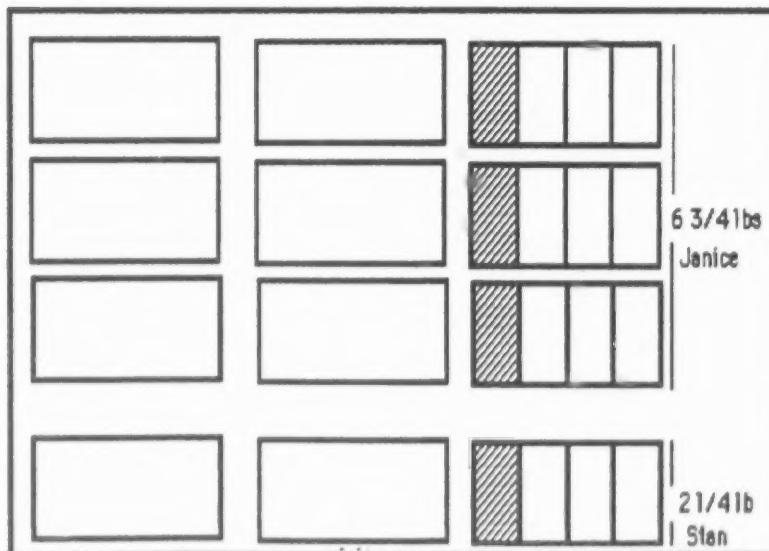
Teacher's Guide

Teacher Procedures: Situation 2. Combine unequal groups by addition.

READ How many pounds of hamburger meat did they buy together?

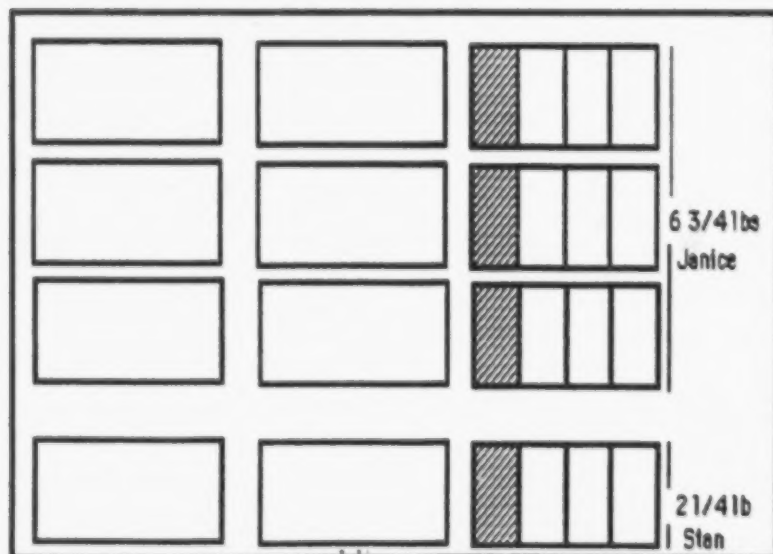
THINK 1. Use concrete models on the transparency to show the two amounts to be combined, Stan's $2\frac{1}{4}$ lbs and Janice's $6\frac{3}{4}$ lbs.

- (a) Ask the students to use their concrete models on the Student Situation Page to show the two amounts which are to be combined. The transparency and the students' pages should be similar to:



- (b) Ask "How many groups are there?" Students should respond "Two, a group of $6\frac{3}{4}$ lbs and a group of $2\frac{1}{4}$ lbs of meat."

2. "How many pounds of hamburger meat did they buy together?" requires that we *combine* the groups. Ask students "Are the groups equal or unequal?" They should respond "They are unequal." Ask the students to combine the two groups on their pages.
3. Careful attention will now be given to move the students from the concrete level of thought through the pictorial level and eventually to the symbolic level using numerals as we combine the two unequal groups.
 - (a) When students understand the action which has taken place using the concrete models, ask them to remove the models and place fraction strips on their pages to represent the two groups of meat. The transparency and their pages should be similar to:



- (b) Ask "How are these groups different?" Students should respond "They are *unequal*." When students understand the problem situation using the models at the concrete level, ask them to remove the models and take out fraction strips to represent

(c) When students understand the combining action using the pictorial representation, instruct them to draw fraction strips to represent the $6\frac{3}{4}$ lbs of meat bought by Janice and the $2\frac{1}{4}$ lbs of meat bought by Stan. Their pages should be similar to illustration in Step 3 (a) above.

4. Say "Combine two or more unequal groups by addition."

5. Say "Now write the number sentence to show what has happened to the two groups in this problem."

(b) Instruct students to write the number sentence that will summarize this combining action in the space at the bottom of their pages. The sentence will be: $6 \frac{3}{4} + 2 \frac{1}{4} = \underline{\hspace{2cm}}$

6. Find the answer to the number sentence.

7. Check the computation in one of these ways:

$$\begin{array}{r} 9 \\ - 6 \frac{3}{4} \\ \hline \end{array}$$

2 1/4

98

Teacher's Guide

Teacher Procedures: Situation 3. Combine equal groups by multiplication.

READ If the hamburger meat costs \$1.89 per pound, what is the total cost of the meat?

- THINK**
1. Display the transparency showing the 9 packages of hamburger meat each costing \$1.89.
 - (a) Instruct students to display pictures of the 9 packages of hamburger meat.
 - (b) Ask "Does each package cost the same amount?" Students should respond that each package costs \$1.89. Their pages and your transparency should be similar to:



2. "What is the total cost of the meat?" requires that we *combine* the 9 equal groups of \$1.89.
3. Careful attention will be given to move the students to the symbolic level of understanding as we combine the 9 equal groups. Say "The question of the problem is 'What is the total cost of the hamburger meat?'"

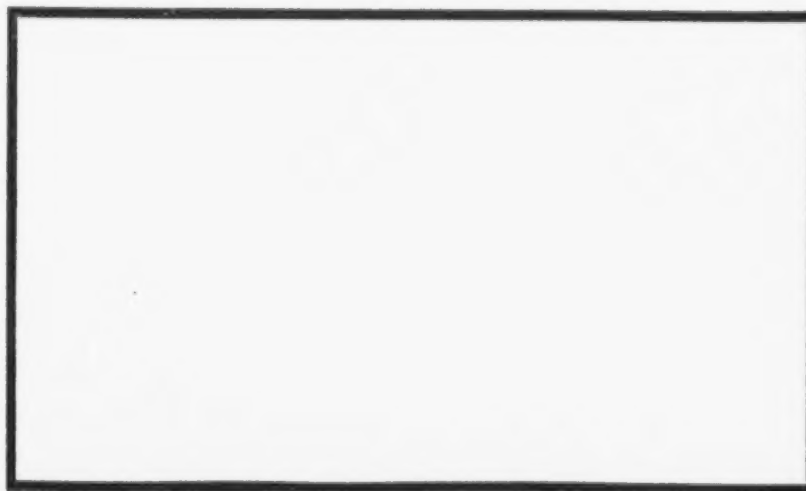
Situation 3: Combine equal groups.

What is the total cost of the 9 pounds of hamburger meat?



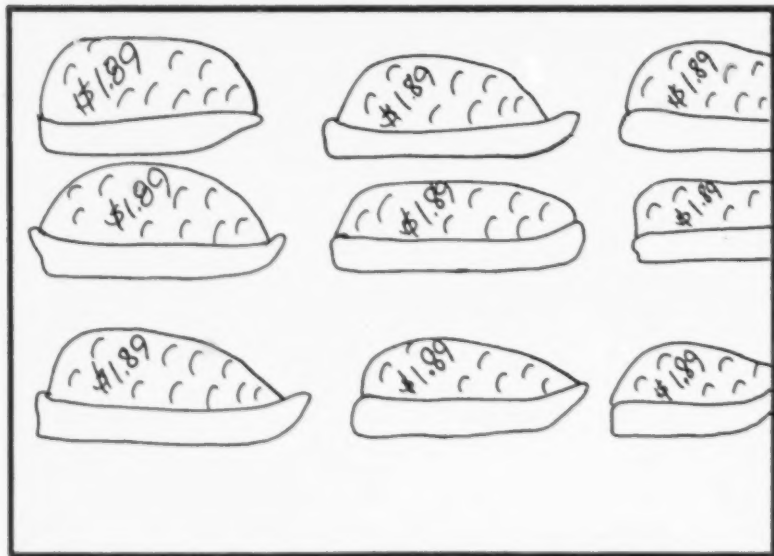
Combine the groups.

Draw the groups.



Write the number sentence: _____

- (a) Ask "How much does one pound of meat cost?" Students should respond "\$1.89." Encourage students to touch each one pound group and say "\$1.89" for each pound of meat. Ask "How many times was \$1.89 spent?" Students should respond "Nine times."
- (b) When students understand the problem situation, *combine equal groups*, ask them to draw nine packages of meat and label each one \$1.89. Their pages and the transparency should be similar to:



- (c) When students understand this pictorial representation accompanied by the numerical price of each pound of meat, ask students to draw a ring around the entire group to indicate the combining action. Say "We have nine groups that are all priced the same amount. *Combine equal groups by multiplication.*"

DECIDE

4. Say "We need to combine nine groups that are equal. *When combining equal groups, multiply.*"

WRITE

5. Say "Now write the number sentence to show what has happened to the nine groups in this part of the problem."

(a) Say "Combine equal groups by multiplication"

(b) Instruct students to write the number sentence that will summarize this combining action in the space at the bottom of the page. The sentence will be: $9 \times \$1.89 = \underline{\hspace{2cm}}$

SOLVE

6. Find the answer to the number sentence.

$9 \times \$1.89 = \$17.01$. The total cost of the 9 pounds of hamburger meat bought by Janice and Stan was \$17.01.

CHECK

7. Check the computation in one of these ways:

$\$17.01 \div 9 = \$1.89$ or $\$1.89 \times 9 = \$17.01$ or

$\$1.89$
1.89
1.89
1.89
1.89
1.89
1.89
1.89
1.89
\$17.01

Additional Practice

Encourage students to pose other problem involving purchases and amounts of money. Some examples you might suggest are:

- (1) Jimmy bought 2 lbs of apples at \$.59 per pound. Billy bought $3\frac{1}{2}$ lbs of oranges which sold for \$.79 per pound. Who spent more money, Jimmy or Billy? How much more money?

(2) Uncle George planted $10 \frac{3}{4}$ lbs of butter bean seeds, 2 lbs of cucumber seeds, and $22 \frac{1}{2}$ lbs of corn seeds in his summer garden. How much money did he spend in all? Which vegetable seeds cost more for the total amount bought of that type seed? The seeds were priced:

butter bean seeds	\$1.19 per pound
cucumber seeds	\$.89 per pound
corn seeds	\$1.50 per pound

Have the students write word problems related to marketing for groceries. A good stimulus might be having the grocery advertisements from the newspaper available for handy reference. Ask the student to draft problems individually.

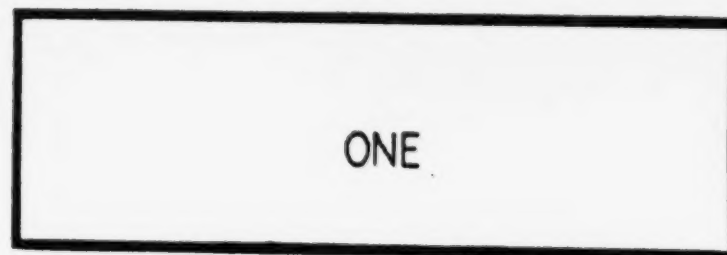
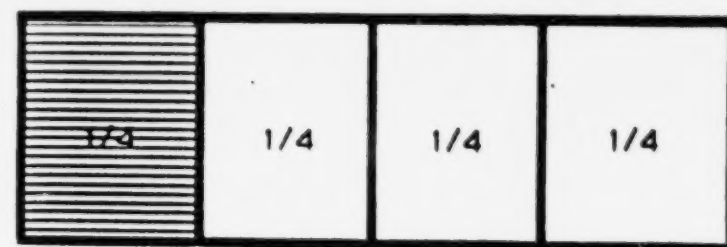
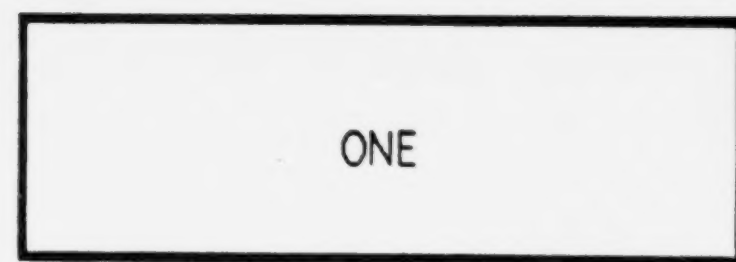
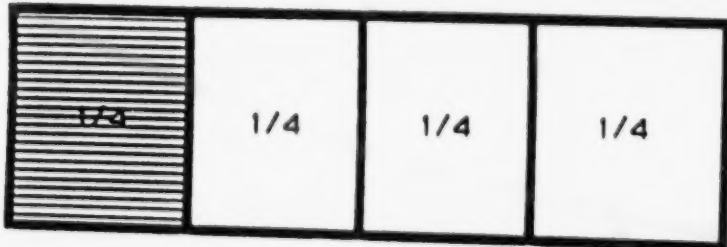
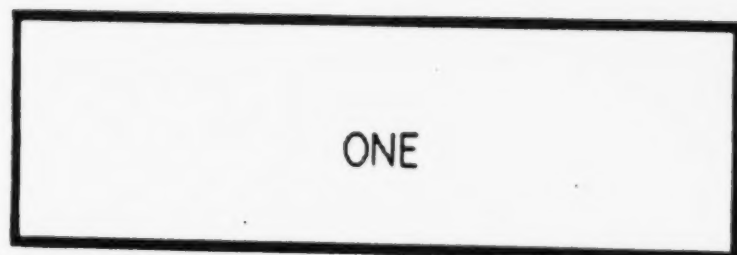
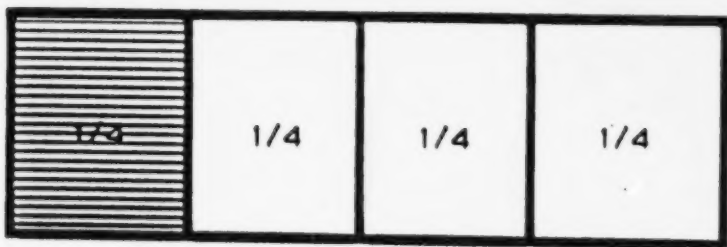
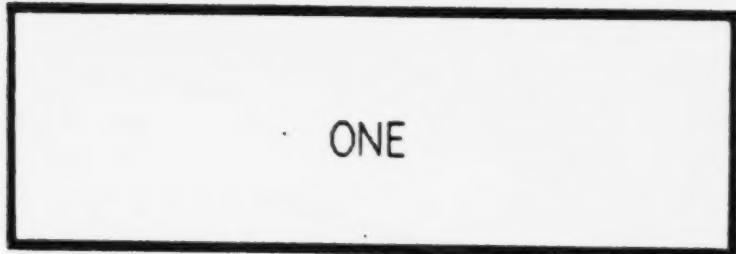
Each student should then read his/her problem to determine that:

- (1) the problem contains a mathematical situation
- (2) the problem contains the necessary information
- (3) the question is clearly stated, and
- (4) the problem is presented in his/her best composition form.

Then, have the students exchange their problems with peers in the classroom. The second student is to review the problem and solve it if all the necessary information is available. Should it not be available, the problem is returned to the author for revision. Finally, when it has been determined that the draft material is well written, ask the student to rewrite the problem for publication either by displaying on the bulletin board, placing in a special problem solving box, exchanging with classrooms of the same grade level, or by using some other strategy for sharing written word problems.



Situation Pictures



Grade 6



Lorenzo and Pete decided to build a patio onto Uncle George's house. They would need to mix the mortar using 1 part cement to 2 parts sand to 3 parts gravel. They would use the mortar to hold the 200 bricks and 85 stepping stones firmly together. If Lorenzo ordered 2 sacks of cement, how much sand would he need? What is the ratio of cement to sand? How much gravel would Lorenzo need to order if he used 3 parts of cement?

Situation: Compare two groups to find a ratio.

Box A	Box B

Write the number sentence for the ratio between the two groups.

Teacher's Guide

Grade Level: Grade Six

Objective: Each student will develop an understanding of compare two or more groups to find a ratio by dividing.

Prerequisite Skills: 1. Understanding of basic addition, subtraction, multiplication, and division facts.

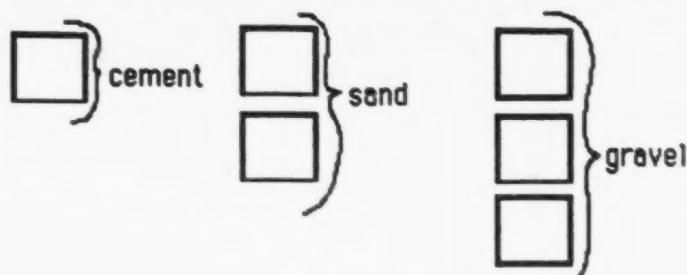
Teacher Guidelines:

1. Model and describe the situation presented on the student's page.
2. Manipulate the display and discuss the various situations and groups involved.
3. Analyze the situation presented.
4. Identify the number of groups.
5. Consider the assumed actions(s) and the question of the problem posed.
6. Write the appropriate number sentence to describe the action of the situation.
7. Select a second problem situation to model and repeat steps 1 - 6.
8. Pose other possible problems for consideration and repeat the above procedures.

Teacher procedures: Situation. Compare two or more groups to find a ratio by dividing.

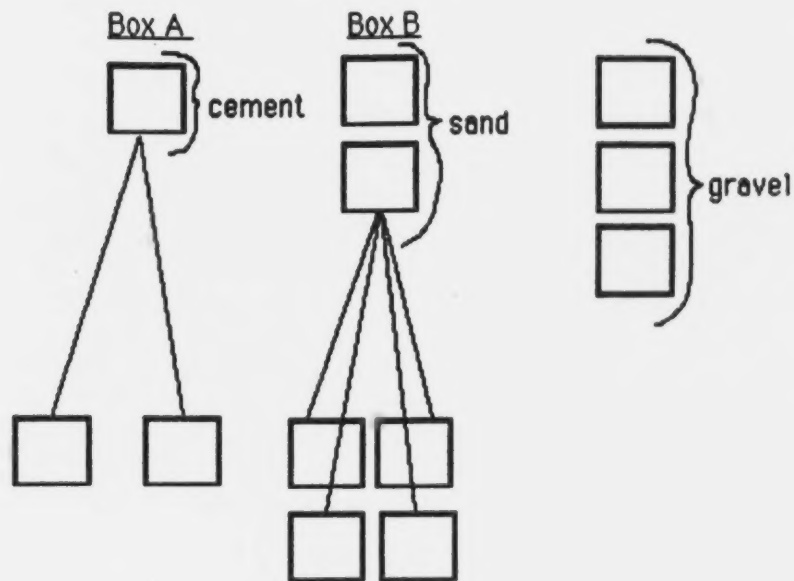
READ Lorenzo and Pete decided to build a patio onto Uncle George's house. They would need to mix the mortar using 1 part cement to 2 parts sand to 3 parts gravel. They would use the mortar to hold the 200 bricks and 85 stepping stones firmly together. If Lorenzo ordered 2 sacks of cement, how much sand would he need?

- THINK 1. Use the transparency of the student's Situation Page to model the problem. Use centimeter blocks or small cubes to represent the ratio of cement to sand to gravel by placing the models on top of the picture. The display will be similar to:



- (a) Instruct students to use manipulative pieces to match the actions that you demonstrate on the overhead projector.
 - (b) Ask "What are the stated amounts of cement, sand, and gravel in the mortar mixture? Students should respond "There is one part cement to two parts sand to three parts gravel. Encourage students to point to each part of the mixture as they say the ratio.
 - (c) Ask "How many groups are in this comparison situation?" Students should respond "There are three groups: the cement, the sand, and the gravel."
2. Ask "If Lorenzo ordered 2 sacks of cement, how much sand would he need?" To answer this question, ask students to refer to the models on their pages pictured above. Say "The stated ratio between cement and sand is 1 to 2. Students must assume that if the amount of one part of the ratio is changed, the other parts of the ratio will change in like manner. So, if the cement is increased by two, the amount of sand will also increase by two."
3. Careful attention will now be given to move students from the concrete level of thought through the pictorial level and eventually to the symbolic level using numerals as two groups are compared to find a ratio.

- (a) Instruct students to place 2 cubes to represent the cement in Box A and 4 cubes to represent the sand in Box B. The display on the transparency and on the student's pages should be similar to the following diagram:



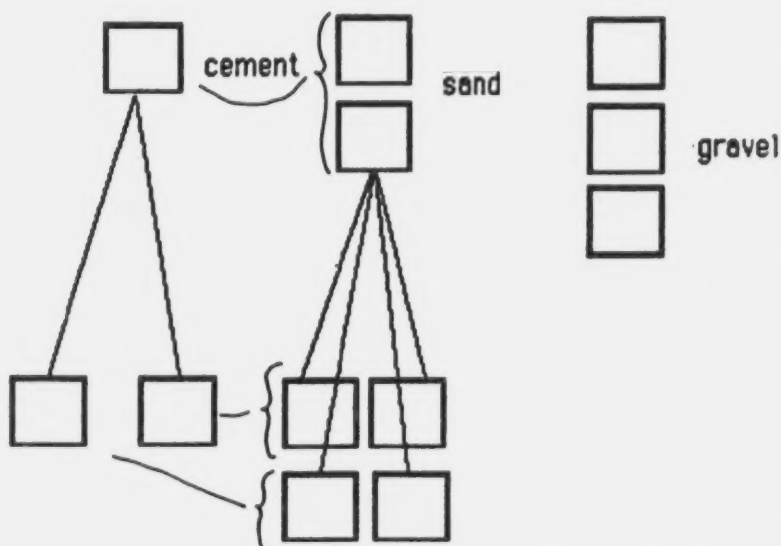
- (b) When students understand the relationship between the amounts of cement and sand using the concrete models, ask them to remove the cubes from their sheets and replace them with the paper cutouts of centimeter cubes. These pictures will represent the amounts of the building materials at a semi-concrete level of understanding.

Place 2 pictures of cubes in Box A and 4 pictures of cubes in Box B. The display will be similar to Step 3 (a) above.

- (c) When students have displayed pictures to represent the ratio and understand the ratio at the pictorial level, ask them to replace the pictures with 2 "X" marks to represent the cement and 4 "X" marks to represent the sand. Write the corresponding numerals to summarize the ratio. The student pages and the overhead transparency will be similar to:

<u>Box A</u>		<u>Box B</u>		
<u>X</u> cement	to	<u>XX</u> sand	=	1 to 2
<u>XX</u> cement	to	<u>XXXX</u> sand	=	2 to 4

- DECIDE 4. Say "We have stated a ratio between cement and sand based on increasing the proportions of parts of the mixture. That ratio is 2 parts cement to 4 parts sand. Division will allow us to determine if a ratio is stated in simplest terms. The ratio 2 to 4 can be divided by the greatest common factor 2 to obtain the original ratio, 1 to 2.
- WRITE 5. The ratio 2 parts cement to 4 parts sand is stated 2 to 4.
- SOLVE 6. Lorenzo would need to buy 4 parts of sand if he increased the amount of cement to 2 parts.
- CHECK 7. Match the blocks representing the cement to cement and sand to sand in one-to-one correspondence for those students who do not understand that the ratio of 2 to 4 is the same as 1 to 2.



Additional Practice

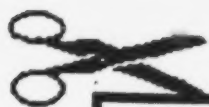
Encourage students to pose problems for group solution, discuss and manipulate the numerical values with the cubes, pictures, "X" marks, and finally the numbers themselves. These important activities will help to bridge the transition from the concrete level of reasoning to the abstract level of understanding, a vital skill in today's societal uses of mathematics.

For the more difficult ratio problems which follow, try allowing students to work in small groups and combine their concrete models and pictures of cubes.

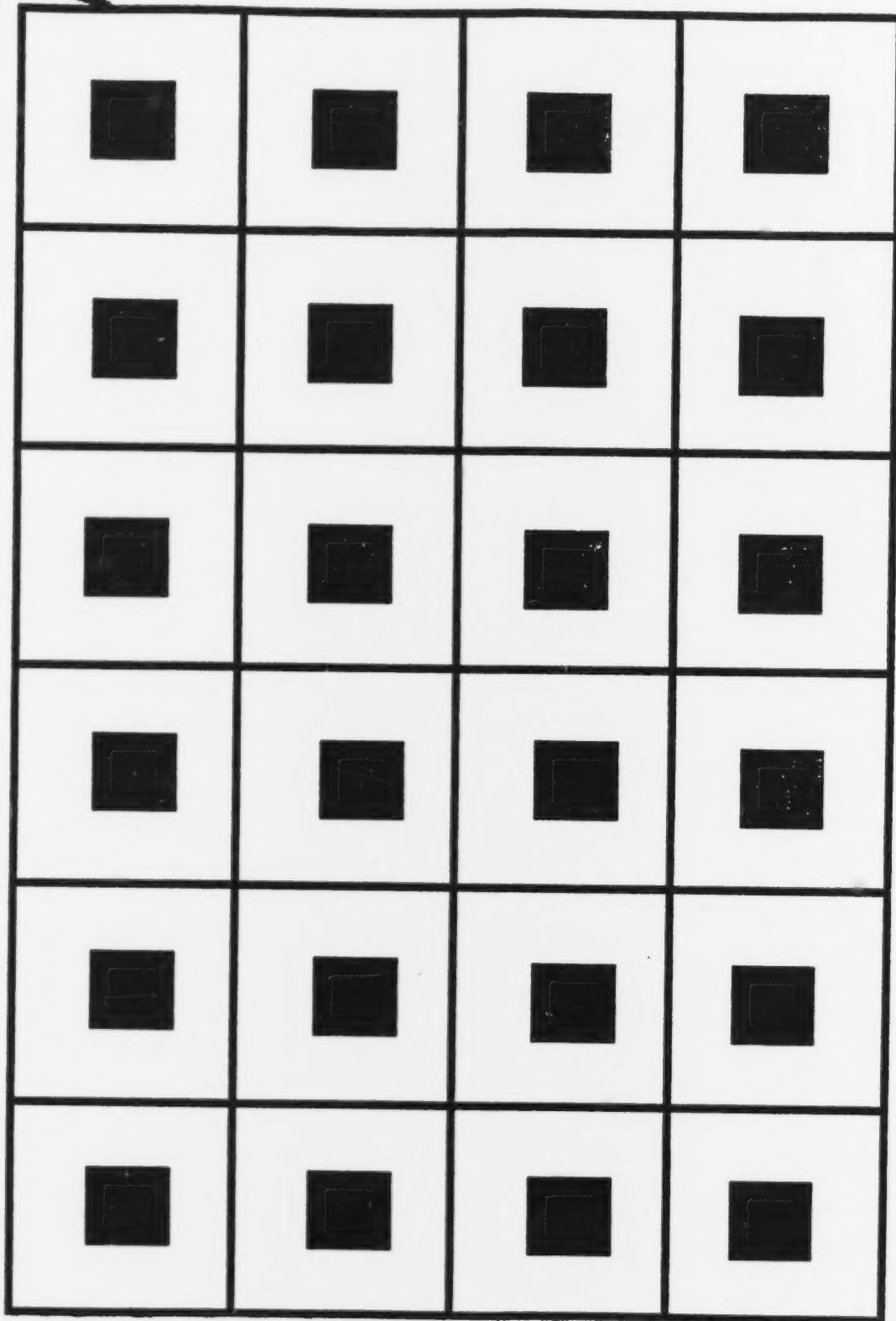
- (1) For every 10 stepping stones that were used in the sidewalk, 25 lbs of mortar mix was required. If the school sidewalk consisted of 70 stepping stones, how much mortar mix was used?
- (2) Every 10 feet of the sidewalk required 8 stepping stones. If the sidewalk was 185 feet long, how many stepping stones were used?
- (3) A brick wall was constructed across the back of the school yard. If 200 brick were used every 3 feet of length, how many brick were used in the 1200 ft. length wall?

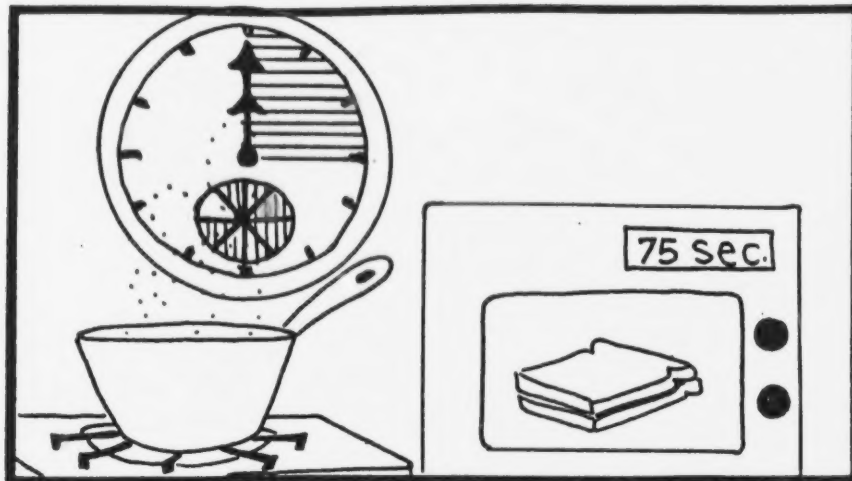
Students should be encouraged to write their own word problems when they have developed an understanding of the action *comparing groups to find a ratio*. A sequence the teacher might choose to follow is:

- (1) Display several groups of cubes on the overhead projector. Ask students to write a problem about amounts of building materials represented by the cubes.
- (2) When the class has finished drafting their own problems, have each student examine his/her word problem carefully to see that the necessary information has been given to the reader. Students should also proofread to determine if the question asked will lead the reader to compare groups to find a ratio.
- (3) Have the students exchange their problems and attempt to solve those written by their peers. Errors which are identified should be corrected by the author.
- (4) Final drafts should be created and the problems published through displaying them on the bulletin board, placing them in a special problem solving box, exchanging with classrooms of the same grade level, or some other strategy for sharing.



Situation Pictures





Jefferson stopped work for a quick lunch break. He had 15 minutes before his next appointment at the automotive shop. He decided to make a cup of instant soup and heat a cheese sandwich in the microwave. If it took $\frac{7}{8}$ of a minute to boil the water for the soup and 55 seconds to heat the cheese sandwich, how much time would he have to eat after preparing his lunch? Will it take longer to boil the water or heat the sandwich? How much longer?

Situation 1: Separate a group into parts.

Write the number sentence. _____

Teacher's Guide

Grade Level: Grade Seven

Objective: Each student will practice to maintain his/her knowledge of combining groups, comparing groups, and separating one group into parts.

Prerequisite Skills:

1. Understanding of seconds, minutes, and hour relationships.
2. Multiplication with common fractions.

Teacher Guidelines:

1. Model and describe the situation presented on the student's page.
2. Manipulate the display and discuss the various situations and groups involved.
3. Analyze the situation presented.
4. Identify the number of groups.
5. Consider the assumed action(s) and the question of the problem posed.
6. Write the appropriate number sentence to describe the action of the situation.
7. Select a second problem situation to model and repeat steps 1 - 6.
8. Pose other possible problems for consideration and repeat the above procedures.

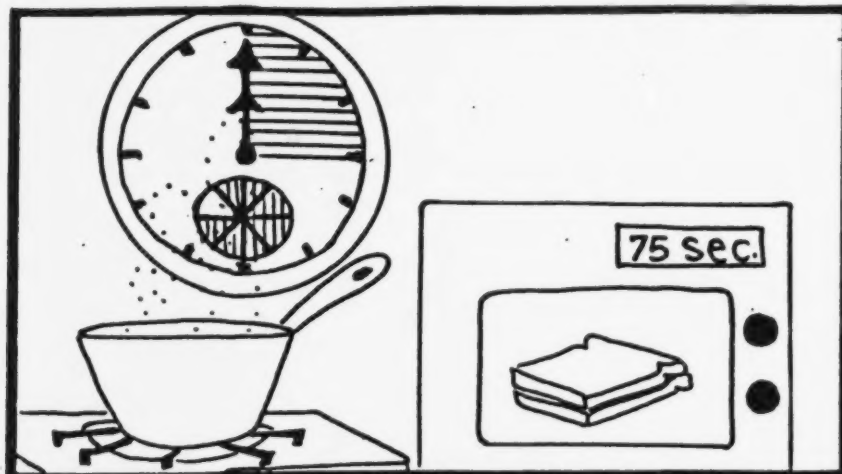
Teacher Procedures: Situation 1, Separate a group into parts.

READ

Jefferson stopped work for a quick lunch break. He had 15 minutes before his next appointment at the automotive shop. He decided to make a cup of instant soup and eat a cheese sandwich. If it took $\frac{7}{8}$ of a minute to boil the water for the soup and 55 seconds to heat the cheese sandwich in the microwave, how much time would he have to eat after preparing his lunch?

THINK

1. Use the transparency of the student's page and place manipulative objects to represent the fractional amount of a minute and 55 seconds.



- (a) Instruct students to display concrete models of the groups presented in the problem.
- (b) Ask "How many amounts of time will we work with in this problem?" Students should be able to determine there is one group of $\frac{7}{8}$ of a minute required to boil water for soup, one group of 55 seconds needed to heat a cheese sandwich, and a group of 15 minutes which represents the time of the lunch break.
- (c) Ask students to touch the models of $\frac{7}{8}$ minute, 55 seconds, and 15 minutes to realize how different they are from each other. Students should recall from earlier experiences in mathematics that operations cannot be performed, nor actions taken, on units which are unlike as the three groups are in this problem.

2. The problem stated that Jefferson had a 15 minute lunch break during which time he would both *prepare* and *eat* his lunch. This situation raises a very subtle assumption that students must consider. They have to assume that the $7/8$ minute and 55 seconds *together* account for the full amount of food preparation time. All the time remaining after these two groups are taken out of the 15 minute lunch period will be spent eating.
3. Ask "What action is required to determine the time needed to prepare lunch?" Students should respond that the time to boil water for soup, $7/8$ of a minute, and the time to heat the sandwich, 55 seconds, must be combined. Before this action can be taken, however, students should realize that the two amounts require a common denominator. When the two amounts of time have been combined, we are asked to remove this amount from the total time of the lunch break. To answer these questions, two actions are required:

- (a) *Combine two or more unequal groups by addition*
- (b) *Separate one group into parts by subtraction*

DECIDE

4. When students understand the actions which must occur to answer the questions of the problem, determine a common denominator for the two amounts of cooking time.
 - (a) Students must depend on past learning to recall that one minute is equal to 60 seconds.
 - (b) Since one group, 55 seconds to heat the sandwich, is already expressed in seconds, convert the $7/8$ minutes to seconds. Since we need to find $7/8$ of a minute, multiply $7/8$ times 60 sec: $7/8 \times 60 = 52.5$ sec.

WRITE

5. Write the number sentences to (a) combine unequal groups, and (b) separate a group into parts.

(a) $52.5 \text{ sec} + 55 \text{ sec} = \underline{\hspace{1cm}} \text{ sec}$

(b) $15 \text{ min} - (52.5 \text{ sec} + 55 \text{ sec}) = \underline{\hspace{1cm}} \text{ min}$

SOLVE

6. (a) Say "Combine the two groups, 52.5 sec and 55 sec, by addition as the groups are *unequal*."

$$\begin{array}{r} 52.5 \text{ sec} \\ +55.0 \text{ sec} \\ \hline 107.5 \text{ sec} \end{array} = \text{combined cooking time for boiling water for the soup and heating the cheese sandwich.}$$

- (b) Say "Separate the one lunch break, 15 minutes, into parts. In other words, separate the cooking time, 107.5 sec out of the 15 minutes. Again a problem arises when the two amounts are in unlike term. There are two ways in which to express the two groups in like terms: (1) convert both cooking times to seconds and then convert 15 minutes to seconds. Separate the cooking time from the total lunch time and then change the result back to minutes; or (2) convert all figures to minutes from the beginning of the computation.

- (1) Convert 15 minutes to seconds:

$$15 \text{ minutes} \times 60 \text{ sec} = 900 \text{ sec in the lunch break}$$

Say "Now separate 107.5 sec from 900 sec."

$$\begin{array}{r} 900.0 \text{ sec} \\ -107.5 \text{ sec} \\ \hline 792.5 \text{ sec} \end{array}$$

$$792.5 \text{ sec} / 60 \text{ sec} = 13 \text{ min } 21 \text{ sec}$$

Jefferson will have 13 min 21 sec to eat lunch.

(2) Convert 107.5 sec to minutes:

$$107.5 \text{ sec} / 60 \text{ sec} = 1.79 \text{ min}$$

Separate 1.79 min from the 15 min lunch break:

$$\begin{array}{r} 15.00 \text{ min} \\ - 1.79 \text{ min} \\ \hline 13.21 \text{ min} \end{array}$$

Jefferson will have 13 min 21 sec to eat lunch.

CHECK

7. Check the accuracy of the computation in one of these ways:

$$\begin{array}{r} \text{(a) } 107.5 \text{ sec} \\ - 55.0 \text{ sec} \\ \hline 52.5 \text{ sec} \end{array} \quad \begin{array}{r} 107.5 \text{ sec} \\ - 52.5 \text{ sec} \\ \hline 55.0 \text{ sec} \end{array} \quad \text{or}$$

$$\begin{array}{r} \text{(b) } 13.21 \text{ min} \\ + 1.79 \text{ min} \\ \hline 15.00 \text{ min} \end{array} \quad \begin{array}{r} 15.00 \text{ min} \\ - 13.21 \text{ min} \\ \hline 1.79 \text{ min} \end{array}$$

The computation is correct. Jefferson will have 13 min and 21 sec to eat his lunch after the preparation time.

Student's Situation Page

Situation 2, Compare two groups to find a difference

Box A

Box B

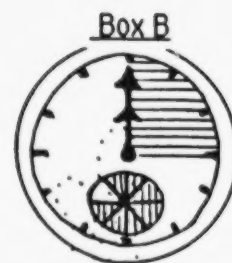
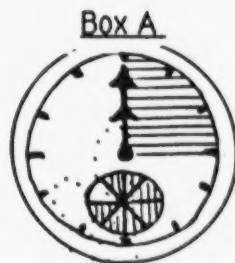
Write the appropriate number sentence.

Teacher's Guide

Teacher Procedures: Situation 2, Compare two groups to find a difference by subtraction

READ Will it take longer to boil water to make the soup or to heat the cheese sandwich? How much longer?

- THINK**
1. Use the transparency of the student's page and concrete objects to represent the time that was required to cook the soup and heat the sandwich. In Situation 1 above, both of these amounts were expressed in like terms.
 - (a) Instruct students to display the groups in terms of seconds using the clock face. Show 52.5 seconds to boil water for the soup and 55 seconds to heat the cheese sandwich. Since both terms are in the same denomination, we can compare them directly.
 - (b) Ask "How many groups are there to compare?"
Students should respond "Two, 52.5 sec and 55 sec."
 2. When the problem was first considered in Situation 1, students had to assume that a difference existed in the two cooking times. Now that these two groups have been expressed in like terms, we can see that they are different in quantity.
 3. When students understand the action which has taken place using concrete representation, ask them to remove the concrete objects and take out the pictures to represent the groups of time at the pictorial level. Use pictures of clock faces to examine the two groups of cooking time: that we must compare: 52.5 sec and 55 sec.
 - (a) Instruct students to place a picture of the clock showing 55 sec in Box A and the picture showing 52.5 sec in Box B. Their pages should be similar to:



- (b) After discussing the differences between these two groups of time, ask students to remove the pictures from their pages and make "X" marks to represent the two groups of time. Their pages should be similar to:

<u>Box A</u>	<u>Box B</u>
X X X X X X X X X	X X X X X X X X X
X X X X X X X X X	X X X X X X X X X
X X X X X X X X X	X X X X X X X X X
X X X X X X X X X	X X X X X X X X X
X X X X X X X X X	X X X X X X X X X
X X X X X	X X X

DECIDE

4. The problem asks that we compare the two groups of time to find a difference. Say "Compare two or more groups to find a difference by subtraction."

WRITE

5. Write the number sentence that will find the difference between two groups:

$$55.0 \text{ sec} - 52.5 \text{ sec} = \underline{\hspace{2cm}}$$

SOLVE

6. Solve the number sentence or fact that will find the difference between two groups.

$$\begin{array}{r} 55.0 \text{ sec} \\ -52.5 \text{ sec} \\ \hline 2.5 \text{ sec} \end{array}$$

The cooking time for the cheese sandwich is 55 sec and the time required to boil water for the soup is 52.5 sec. It will take 2.5 sec longer to heat the sandwich than cook the soup.

CHECK

7. Check the accuracy of the subtraction computation in one of these ways:

$$\begin{array}{r} 52.5 \text{ sec} \\ + 2.5 \text{ sec} \\ \hline 55.0 \text{ sec} \end{array}$$

$$\begin{array}{r} 55.0 \text{ sec} \\ - 2.5 \text{ sec} \\ \hline 52.5 \text{ sec} \end{array}$$

The computation is correct. It will take 2.5 sec longer to prepare the cheese sandwich than to boil water for the soup.

Additional Practice

Encourage students to pose other problems involving fractional amounts. Remember to convert terms that are unlike, such as minutes and seconds and unlike fractions into like terms. For example, try some of the following problems:

- (1) Jane has 1 hour before her guests arrive. The pizza must bake 22 minutes. It will take 35 minutes to prepare all of the ingredients. If her guests arrive $\frac{2}{10}$ of an hour early, will the pizza be ready? Why or why not?
- (2) Jim's race at the motocross was scheduled to start in 25 minutes. He had to add fuel, grease the gears, and adjust the slack in the chain. If it takes $9\frac{1}{2}$ minutes to add fuel, $10\frac{2}{3}$ minutes to grease the gears, and 4 minutes to adjust the chain, will he be late for the race? If not, how much time will he have to spare?

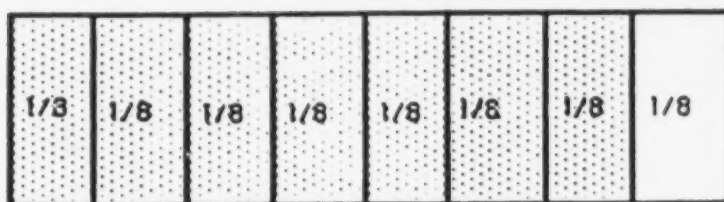
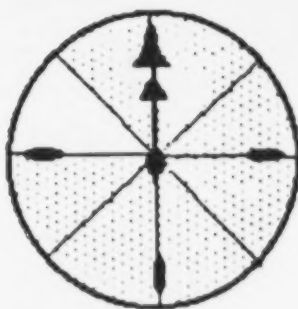
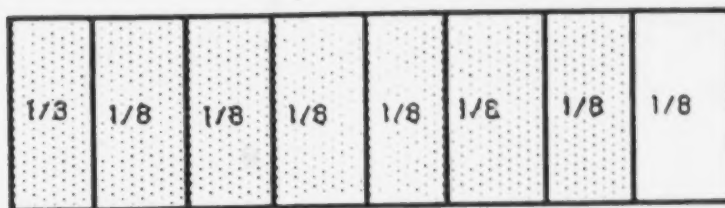
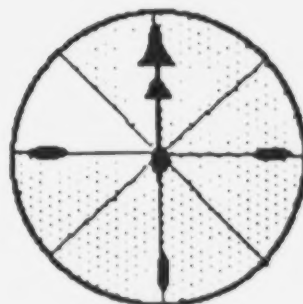
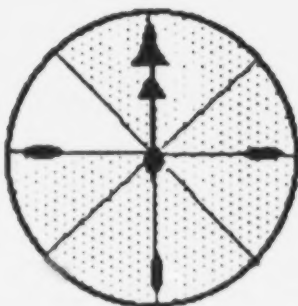
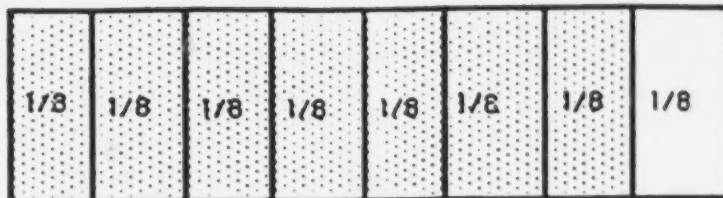
- (3) The recipe for oatmeal cookies called for $1\frac{1}{2}$ cups of sugar, $\frac{3}{4}$ cup of shortening, $\frac{1}{4}$ cup water, 1 tsp vanilla, 3 cups oats, 1 cup flour, 1 tsp salt, and tsp soda. Jane needed to double the recipe for the party. She took down the sack of flour and measured 2 cups. But, she turned over the measuring cup and spilled $\frac{1}{4}$ of the flour on the floor. When she measured a second time, there was $2\frac{1}{3}$ cups of flour remaining in the sack. Will there be enough flour to make the doubled recipe of oatmeal cookies this time? If so, how much flour will remain in the sack? Will she have to buy another sack of flour before the cookies can again be made?

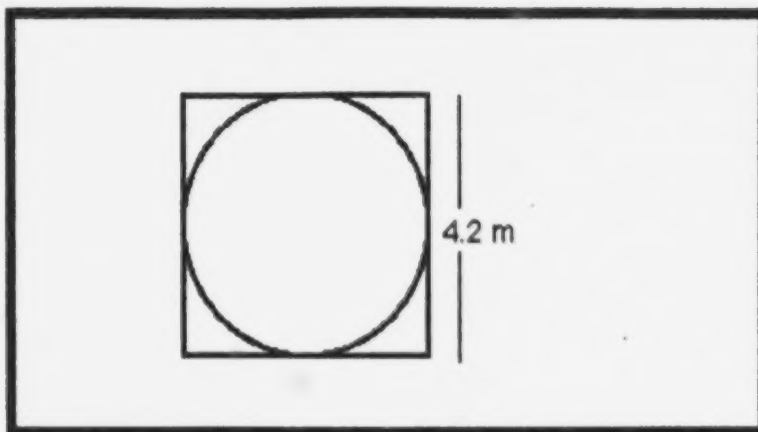
Students should be encouraged to write their own word problems when they have developed an understanding of the action of multi-step word problems. A sequence the teacher might choose to follow is:

- (1) Display several recipes about the room. Ask the student to write a problem about these recipes which requires at least two different procedures prior to attaining the answer of the question of the problem.
- (2) When the members of the class have finished drafting their own problems, have each student examine his/her word problem carefully to see that the necessary information has been given to the reader. Students should also proofread to determine that the question asked will lead the reader to use a multi-step problem solving process.
- (3) Have the students exchange their problems and attempt to solve those written by their peers. Errors which are identified should be corrected by the author.
- (4) Final drafts should be created and the problems published through displaying them on the bulletin board, placing them in a special problem solving box, exchanging with classrooms of the same grade level, or some other strategy for sharing.



Situation Pictures





What is the difference between the circumference of a circle and the perimeter of a square that touches the circle on the outside? The side of the square is 4.2 m.

Situation 1: Compare to find a difference between two groups

Find the circumference of the circle. Find the perimeter of the square.

Compare to find the difference between the two amounts. Which is larger? Which amount is smaller?

Write the appropriate number sentence to *compare* the two amounts.

Teacher's Guide

Grade Level: Grade Eight

Objective: Each student will apply his/her knowledge of geometry formulae in a problem solving setting which requires the action of compare two groups to find a difference by subtraction.

Prerequisite Skills:

1. Understanding of formula for circumference of a circle.
2. Understanding of formula for perimeter of polygons.

Teacher Guidelines:

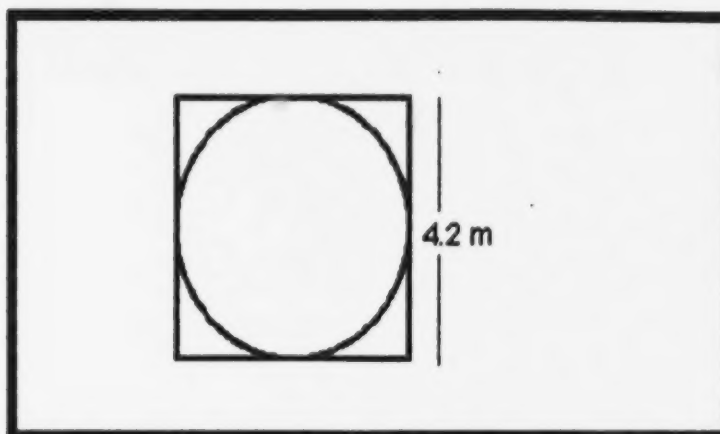
1. Model and describe the situation presented on the student's page.
2. Manipulate the display and discuss the various situations and groups involved.
3. Analyze the situation presented.
4. Identify the number of groups.
5. Consider the assumed action(s) and the question of the problem posed.
6. Write the appropriate number sentence to describe the action of the situation.
7. Select a second problem situation to model and repeat steps 1 - 6.
8. Pose other possible problems for consideration and repeat the above procedures.

Teacher Procedures: Situation: Compare two groups to find a difference.

READ What is the difference between the circumference of a circle and the perimeter of a square that touches the circle on the outside? The side of the square is 4.2 m.

THINK

1. Use the transparency of the student's Situation Page and place a model of a circle and a square of relative sizes in the diagram. Ask students "How can I determine the sizes of the models of squares and circles to use? What information is given in the problem that will allow me to make that decision?"



- (a) Instruct students to display the models of squares and circles on their Situation Pages and reflect on the characteristic they must have in common if the circle is to fit inside the square and touch each side of the square.
 - (b) Ask "What must be true about the size of the circle and the square?" Lead students to determine that the diameter of the circle and a side of the square must have the same measurement.
2. The problem makes the assumption that students can find the perimeter of a square if the length of one side is given. The problem further assumes that students can calculate the circumference of a circle. Move the concrete objects around allowing students to handle various sizes of both circles and squares until they reach the generalization that the diameter must measure the same as the side of the square.

DECIDE

3. Say "We have displayed two objects, a square and a circle. What specifically does the problem ask that we compare? It states "What is the difference between the circumference of the circle and the perimeter of the square?" We are asked to *compare two groups (measurements in this case) to find a difference*. This action requires the application of subtraction.
4. The first step in the solution of this problem, therefore, requires that students compute the circumference of the circle and the perimeter of the square and then use the operation of subtraction to compare to find a difference between two groups. Students will have to rely on their past knowledge of geometry or you, the teacher, will need to supply the formulae:

$$\text{Circumference} = \pi d$$

$$\text{Perimeter} = 4s$$

5. Allow students time to find the circumference of the circle and the perimeter of the square. When these two amounts have been determined, a difference can be found between the two calculations that will answer the question posed in the problem.

WRITE

6. The number sentence should be:

$$(4 \times 4.2) - (3.14 \times 4.2) = \underline{\hspace{2cm}}$$

SOLVE

8. $4 \times 4.2 = 16.8 \text{ m}$ $3.14 \times 4.2 = 13.188$

$$\begin{array}{r} 16.800 \text{ m} \\ - 13.188 \text{ m} \\ \hline 3.612 \text{ m} \end{array}$$

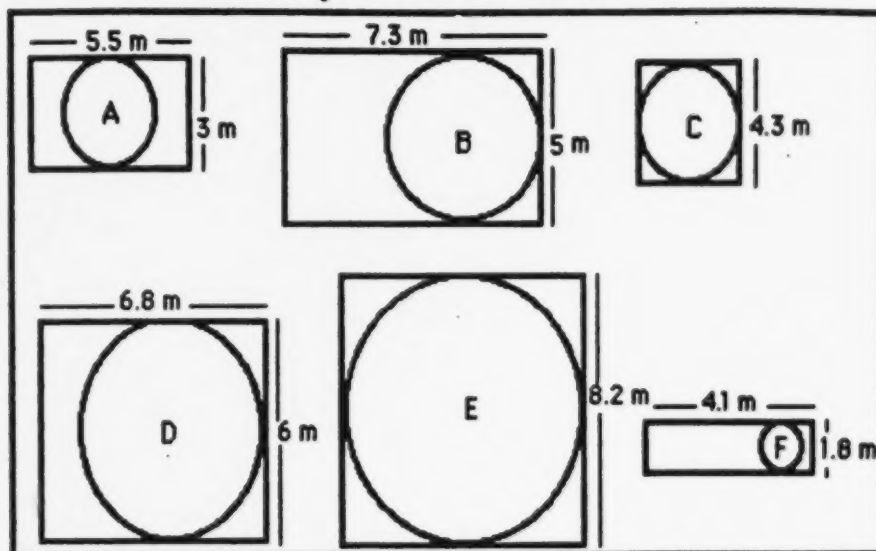
The difference between the perimeter, 16.8 m, and the circumference, 13.188 m, is 3.612 m.

CHECK

9. 13.188 m 16.800 m The computation is correct and
 $+ 3.612 \text{ m}$ $- 3.612 \text{ m}$ the answer is reasonable.
 16.800 m 13.188 m

Additional Practice

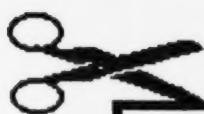
Encourage students to pose problems with other measurements for the side of the square. Also, place a circle inside a rectangle. Remind students that the formula for the perimeter of a rectangle is $P = 2(l) + 2(w)$.



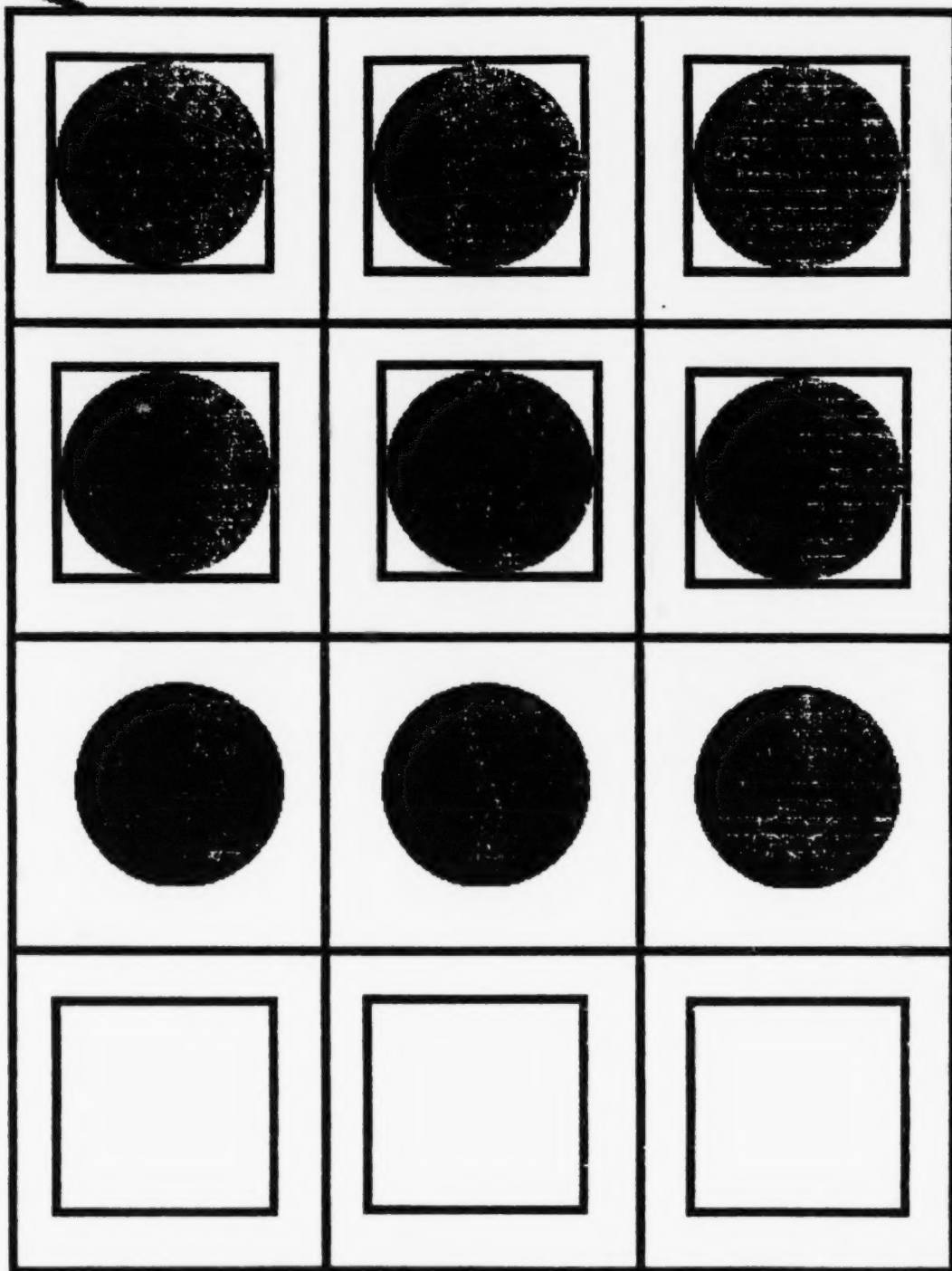
Box	Circumference?	Perimeter?	Difference?
A			
B			
C			
D			
E			
F			

Students should be encouraged to write their own word problems when they have developed an understanding of the action involved in the multi-step problems related to geometry. A sequence that the teacher might choose to follow is:

- (1) Display several different geometric shapes on the overhead projector. Ask each of the students to write a problem about the shapes in which they relate one to another in a multi-step problem.
- (2) When the class has finished drafting their own problems, have each of the students examine his word problem carefully to see that the necessary information has been given to the reader. Students should also proofread to determine that the question asked will lead the reader to solve multi-step word problem related to geometry.
- (3) Have the students exchange their problems and attempt to solve those written by their peers. Errors which are identified should be corrected by the author.
- (4) Final drafts should be created and the problems published through displaying them on the bulletin board, placing them in a special problem solving box, exchanging with classrooms of the same grade level, or some other strategy for sharing.



Situation Pictures



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APPENDIX A

A SCOPE AND SEQUENCE OF TAPS APPLICATIONS

K 1 2 3 4 5 6 7 8

COMBINE

Groups are unequal, Add X X X X X X X X X

Groups are equal, Multiply X X X X X X

SEPARATE

Into parts, Subtract X X X X X X X X

Into equal parts, Divide X X X X X X

COMPARE

To find a difference, Subtract X X X X X X X

To find a ratio, Divide X X X

Grade: K 1 2 3 4 5 6 7 8